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On measurable norms and abstract Wiener spaces

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§1. Introduction

In [4], H. Kuo has shown that the following:

THEOREM A. *Let H be a real separable Hilbert space with norm $\|\cdot\|_H$, and $\|\cdot\|$ be a continuous Hilbertian norm on H. Then the following conditions are equivalent.*

(1) $\|\cdot\|$ *is measurable.*

(2) *There exists a one-to-one Hilbert-Schmidt operator T of H such that $\|x\|=\|Tx\|_H$ for $x\in H$.*

In general, if $\|\cdot\|$ be a measurable norm (not necessarily Hilbertian one), there is a compact operator K of H such that $\|x\|\leqq\|Kx\|_H$ for $x\in H$. The above theorem shows that if a measurable norm $\|\cdot\|$ be a Hilbertian one, then the operator K can be taken to be a Hilbert-Schmidt operator. However, if a measurable norm $\|\cdot\|$ be not a Hilbertian one, then this is not necessarily true. The counterexample can be found in [4].

The purpose of the present paper is to show that under the suitable conditions of the norm $\|\cdot\|$, Theorem A can be extended to a non-Hilbertian case. Throughout the paper, we assume that linear spaces are separable with real coefficients.

§2. Basic definitions and well known results

1°. p-absolutely summing operators and $(*)_p$-conditions $(1\leqq p<\infty)$

Let E and F be Banach spaces.

A sequence $\{x_i\}$ with values in E is called weakly p-summable if for all $x^*\in E^*$, the sequence $\{x^*(x_i)\}\in l_p$.

A sequence $\{x_i\}$ with values in E is called absolutely p-summable if the sequence $\{\|x_i\|\}\in l_p$.

DEFINITION 2.1.1. *A linear operator T from E into F is called p-absolutely summing if for each $\{x_i\}\subset E$ which is weakly p-summable, $\{T(x_i)\}\subset F$ is absolutely p-summable.*

We shall say "absolutely summing" instead of "1-absolutely summing".