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Mean curvatures for certain ν-planes in quaternion Kählerian manifolds

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Introduction.

Let (M, g) be an n-dimensional Riemannian manifold with metric tensor g. We denote by $K(X, Y)$ the sectional curvature for a 2-plane spanned by tangent vectors X and Y at $P\in M$, and by π a ν-plane at $P\in M$. Let $\{e_1, \cdots, e_n\}$ be an orthonormal base of the tangent space at P such that $\{e_1, \cdots, e_\nu\}$ spans π. S. Tachibana [3] defined the mean curvature $\rho(\pi)$ for π by

$$\rho(\pi)=\frac{1}{\nu(n-\nu)}\sum_{b=\nu+1}^{n}\sum_{a=1}^{\nu}K(e_a, e_b),$$

which is independent of the choice of an adapted base for π. He obtained the following

THEOREM A (S. Tachibana [3]). *In an $n(>2)$-dimensional Riemannian manifold (M, g), if the mean curvature for ν-plane is independent of the choice of ν-planes at each point, then*

(i) *for $\nu=1$ or $n-1$, (M, g) is an Einstein space,*

(ii) *for $1<\nu<n-1$ and $2\nu\neq n$, (M, g) is of constant curvature,*

(iii) *for $2\nu=n$, (M, g) is conformally flat.*

The converse is true.

Taking holomorphic 2λ-planes or antiholomorphic ν-planes, instead of ν-planes, analogous results in Kählerian manifolds are also obtained.

THEOREM B (S. Tachibana [4] and S. Tanno [5]). *In a Kählerian manifold (M, g, J) of dimension $n=2l\geqq 4$, if the mean curvature for holomorphic 2λ-plane is independent of the choice of holomorphic 2λ-planes at each point, then*

(i) *for $1\leqq\lambda\leqq l-1$ and $2\lambda\neq l$, (M, g, J) is of constant holomorphic sectional curvature,*

(ii) *for $2\lambda=l$, the Bochner curvature tensor vanishes.*

The converse is true.

THEOREM C (K. Iwasaki and N. Ogitsu [2]). *In a Kählerian manifold (M, g, J) of dimension $n=2l\geqq 4$, if the mean curvature for antiholomorphic*