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Microlocal parametrices for mixed problems for symmetric hyperbolic systems with diffractive boundary

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§ 1. Introduction

Microlocal parametrices for hyperbolic mixed problems in domains with diffractive boundary have been constructed by Ludwig [9], Melrose [11], Taylor [16], Eskin [3] and others for second-order hyperbolic equations with Dirichlet boundary conditions. Taylor [16] (or [18]) has also obtained such results for Neumann boundary conditions, and Imai and Shirota [4] have obtained such results for certain general boundary conditions which include the Neumann conditions. (See also Shirota [14]). Moreover applying the results in [16] Taylor has obtained in [17] such results for Maxwell's equations in the exterior of a strictly convex perfect conductor.

The purpose of this paper is to give a generalization of the above results.

Let Ω be the open half space $\{x=(x', x_n)=(x_0, x'', x_n)\,;\, x_0\in R^1,\ x''\in R^{n-1}, x_n>0\}$ in R^{n+1} $(n\geqq 2)$ with boundary $\partial\Omega$ and $P(x, D)$ a symmetric system of first order defined on $\bar{\Omega}$ which is hyperbolic with respect to x_0. Consider a mixed problem:

$$P(x, D)\,u=\sum_{j=0}^{n}A_j(x)\,D_j u+C(x)\,u=0 \quad \text{in } \Omega\,,$$
$$B(x')\,u=f(x') \quad \text{on } \partial\Omega\,,$$
$$u(x)=0 \quad \text{in } \Omega\cap\{x_0<0\}\,,$$

where $D_j=-i\partial/\partial x_j$, A_j, $j=0, 1, \cdots, n$, are hermitian $m\times m$ matrices, A_0 is positive definite, and A_j, C and B are smooth (i.e., of class C^∞) and are constant for $|x|$ large enough.

Let $f\in\mathscr{E}'(\partial\Omega)$, $f(x')=0$ for $x_0<0$ and the wave front set $WF(f)$ be contained in a conic neighborhood of the diffractive points. We then want to show that there is a parametrix for the mixed problem, i.e., a distribution $u\in\mathscr{D}'(\Omega\cap U)$ with a neighborhood U of sing supp f in R^{n+1} such that $u(x)$ is a C^∞-function of $x_n\geqq 0$ with value in $\mathscr{D}'(R^n_{x'})$ and

(1.1) $\qquad P(x, D)\,u\in C^\infty(\bar{\Omega}\cap U)\,,$