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Module correspondence in finite groups

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1. Introduction

The problem to investigate relations between the local and global properties of a finite group is one of the most important and elementary problems in the theory of finite groups. Concerning a prime number p, several techniques to investigate the global properties from p-local structure of groups are studied by many authors and play an important role for the advancement of the theory of finite groups (see [15]).

The representation theory of finite groups provides an effective tool to connect local properties to global properties. Hence for studies of the representation theory itself, it is also an important to investigate relations between the local and global properties. The result of Brauer and Fowler [5] is a representative example in this direction. They showed that there exist at most a finite number of simple groups with a specified centralizer of an involution. Several induction theorem by Artin, Brauer and others are also representative ones in this problem.

When we are interested in a prime p, the modular representation theory of finite groups with respect to p is a refinement of the ordinary representation theory and at the same time is an effective tool to investigate p-local properties of finite groups. In studying it the theory of vertices and sources of Green, together with a series of studies of Brauer, has many contents to be developed in the future.

In this paper we shall investigate the theory of vertices and sources, especially the Green correspondence and try to refine several results in it for a certain type of finite groups.

In section 2, we are concerned with irreducible modules with trivial sources and investigate properties of the Green correspondence for them.

In section 3, as an application of section 2 we define a family of finite groups called M_p-groups and study the representation theory of them.

In section 4, we are mainly interested in finite solvable groups and some correspondence between a set of irreducible modules of a solvable group and its subgroup. This correspondence relates to the Green correspondence a little.