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A remark on Xia's theorem concerning quasi-invariant measures

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§ 1. Introduction

Let E and F be Banach spaces, and suppose that F itself is a linear subspace of E. Also suppose that the inclusion mapping J of F into E is continuous. Then the author [4, Proposition 3. 1] proved the following:

PROPOSITION 1. 1. *The following implication* (1)$\Rightarrow$(2) *holds.*

(1) *There exists a finite Borel measure on* E *which is quasi-invariant with respect to* F.

(2) *The adjoint mapping* J^* *of* E^* *into* F^* *is absolutely summing.*

In Proposition 1. 1, if E is a Hilbert space and F is a Banach space, then the converse implication (2)$\Rightarrow$(1) also holds (cf. [4, Theorem A]).

Now we shall consider the following problem.

PROBLEM. Can we show the equivalence of statements (1) and (2) of Proposition 1 .1 when E and F belong to some suitable class of Banach spaces?

Concerning this, Xia [7, p. 319, Example 5. 3. 1] and the author [4, Proposition 4. 1. 1] proved the following:

THEOREM 1. 1. *Let* $1\leqq p<\infty$, $1\leqq q\leqq 2$. *If we assume that* $l^q\subset l^p(a_n)$, *then the following statements are equivalent.*

(1) *There exists a finite Borel measure on* $l^p(a_n)$ *which is quasi-invariant with respect to* l^q.

(2) *The adjoint mapping* J^* *of* $(l^p(a_n))^*$ *into* $(l^q)^*$ *is absolutely summing, where* J *denotes the inclusion mapping of* l^q *into* $l^p(a_n)$.

(3) $\sum a_n<\infty$.

The purpose of this note is to give a generalization of the above theorem to a function space, which is stated as follows:

Let (Ω, Σ, μ) and (Ω, Σ, ν) be σ-finite measure spaces. For $1\leqq p<\infty$, $1\leqq q<\infty$, we denote by $L^p(\mu)$ the Banach space of equivalence classes of real valued measurable functions on (Ω, Σ, μ) whose p'th power is integrable,