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A family of difference sets having -1 as an invariant

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A construction is given for difference sets having -1 as an invariant, whose parameters are

$$v=\frac{1}{2}3^{s+1}(3^{s+1}-1),\ k=\frac{1}{2}3^{s}(3^{s+1}+1),\ \lambda=\frac{1}{2}3^{s}(3^{s}+1),\ n=3^{2s}\ (s\ \text{even})\,.$$

Let G be a finite group of order v. A subset D of order k is called a difference set in G with parameters (v, k, λ, n) in case every non-identity element g in G can be expressed in exactly λ way as $g=d_1^{-1}d_2$ with $d_1, d_2\in D$. The parameter n is defined by $n=k-\lambda$. For any integer t, let $D(t)$ denote the image of D under the mapping $g\to g^t$, $g\in G$. If the mapping is an automorphism of G and $D(t)$ is a translate of D, then t is called a multiplier of D. But even if it is not an automorphism, $t=-1$ has an important property, that is, it makes a non-direct graph which has a regular automorphism.

In this paper, we will show an infinite series of difference sets having -1 as an invariant. SPENCE [1] showed a family of difference set with parameters

$$v=\frac{1}{2}3^{s+1}(3^{s+1}-1),\ k=\frac{1}{2}3^{s}(3^{s+1}+1),\ \lambda=\frac{1}{2}3^{s}(3^{s}+1),\ n=3^{2s}\,.$$

By modification of his argument, we will prove the following theorem.

THEOREM. *There exists a difference set with parameter*

$$v=\frac{1}{2}3^{s+1}(3^{s+1}-1),\ k=\frac{1}{2}3^{s}(3^{s+1}+1),\ \lambda=\frac{1}{2}3^{s}(3^{s}+1),\ n=3^{2s}$$

which has -1 as an invariant for each even integer $s\geq 2$.

PROOF. Let E denote the additive group of $GF(3^{s+1})$ and K_1 be the multiplicative group of $GF(3^{s+1})$, where s is an even integer ≥ 2. Then since s is even, we have $K_1=Z/2Z\times K$ for a subgroup K of odd order. Set $G=E*K$ be the semi-direct product of E by K. Then we have the following;