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Convexity in Musielak-Orlicz spaces

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Summary. Some criterion for uniform convexity of a modular I of Musielak-Orlicz type is given. Moreover, some lower estimates are given for the moduli of convexity in uniformly convex Orlicz spaces.

0. Introduction. In paper [4] a criterion is given for uniform convexity under Luxemburg norm of Musielak-Orlicz spaces of vector-valued functions in the case of an atomless measure. In this paper a criterion is given for uniform convexity of a modular I of Musielak-Orlicz type also in the case of an atomless measure. Moreover, in section 2, some lower estimate are given for the moduli of convexity in uniformly convex Orlicz spaces in the case of an atomless as well as a purely atomic measure.

It is well known (see [8] and [13]) that if an Orlicz function satisfies suitable condition Δ_2, then there exist functions δ and η mapping the interval $(0, 1)$ into itself such that the inequalities $I(x)\leq 1-\varepsilon$ and $I(x)\leq\eta(\varepsilon)$ imply $\|x\|_{\Phi}\leq 1-\delta(\varepsilon)$ and $\|x\|_{\Phi}\leq\varepsilon$, respectively (for definitions of functionals I and $\|\ \|_{\Phi}$ see below).

However, for a lower estimate of the modulus of convexity of the Luxemburg norm in Orlicz spaces we need to known some lower estimates of the functions δ and η and of the modulus of convexity δ_I. A lower estimate of the modulus δ_I follows from results of A. Kamińska [8]. Some estimates of the functions δ and η, which yield an estimate of the modulus of convexity of the norm $\|\ \|_{\Phi}$, are main purpose of section 2 of this paper.

R. P. Maleev and S. L. Troyanski [11] gave a best in some sense estimate of the modulus of convexity of the norm $\|\ \|_{\Psi}$, where Ψ is an Orlicz function equivalent to Φ (two equivalent Orlicz functions Φ and Ψ yield the same Orlicz space with equivalent norms $\|\ \|_{\Phi}$ and $\|\ \|_{\Psi}$, see [12]). This problem was continuated by T. Figiel [2] (see e. g. Propositions 19, 21 and Lemma 20). However, for a pair of equivalent Orlicz functions Φ and Ψ an estimate of the modulus of convexity for the norm $\|\ \|_{\Psi}$ need not yield of an estimate of the modulus of convexity for the norm $\|\ \|_{\Phi}$. So, the problem investigated in section 2 of this paper is sensible.

Now, we shall introduce some denotations and definitions. Let $\mathbf{R}$ be the set of real numbers and $\mathbf{N}$ the set of natural numbers. Let $(X, \|\ \|)$ be a real Banach space. By (T, Σ, μ) we denote a space of a non-negative,