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H-separable Extensions and Torsion Theories

In memory of Professor Akira Hattori

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Introduction. Let A be a ring with identity and B a subring of A with common identity. We shall say that A is *H-separable* over B if $A\otimes_{B}A$ is isomorphic to a direct summand of a finite direct sum of copies of A as (A, A)-bimodules. Let C be the center of A and $V_{A}(B)$ the commutator of B in A. Then it is well-known that A is H-separable over B iff the maping η : $A\otimes_{B}A\rightarrow \mathrm{Hom}_{C}(V_{A}(B), A)$ given by $\eta(a\otimes a')(v)=ava'$ for a, a' in A and v in $V_{A}(B)$ is an isomorphism and $V_{A}(B)$ is a finitely generated projective C-module [7, Theorem 1.1].

Recently K. Sugano [8] has pointed out that H-separable extensions of B have close connections with Gabriel topologies on B. He showed, among other things, that if A is left flat and H-separable over B then $V_{A}(V_{A}(B))$ is isomorphic to the localization of B with respect to the right Gabriel topology consisting of all right ideals $\mathfrak{b}$ of B such that $\mathfrak{b}A=A$, where $V_{A}(V_{A}(B))$ denotes the double commutator of B in A. Using this he then showed that if A is H-separable over B and B is regular then $B=V_{A}(V_{A}(B))$.

Motivated by his results we shall study in this paper H-separable extensions of B from the point of view of torsion theories. We shall begin with the study of the torsion class

$$T=\{M_{B}\mid M\otimes_{B}A=0\}$$

of mod-B. If ${}_{B}A$ is flat, then T is hereditary. This assumption, however, is not necessary for T to be hereditary. We shall introduce the notion of weakly flat B-modules and show that the weakly flatness of A ensures T to be hereditary. We shall provide an example to show that not all weakly flat modules are flat. It is shown in case A is H-separable over B a necessary and sufficient condition for $B\rightarrow V_{A}(V_{A}(B))$ to be a right flat epimorphism (Theorem 3.9) and also one for $B=V_{A}(V_{A}(B))$ to hold (Theorem 3.12).

We shall use M_{B} to denote a right B-module M and $M'\leqq M$ a submodule M' of M. Consequently $\mathfrak{a}\leqq B_{B}$ means that $\mathfrak{a}$ is a right ideal of B. For undefined notions about torsion theory we shall refer to [6]. For a right