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A certain logmodular algebra and its Gleason parts

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Abstract. Let A be a weak-* Dirichlet algebra on a nontrivial probability measure space $(X, \mathscr{A}, m)$ and let $H^\infty = H^\infty(m)$ be the weak-* closure of A in $L^\infty(m)$. The first objective of this paper is to study the maximal ideal space $M(H^\infty)$ of H^∞ with a special regard to the algebraic direct sum decomposition $H^\infty = \mathscr{H}^\infty \oplus I^\infty$, where I^∞ is an ideal of H^∞ apeared in [14].

The second objective of this paper is to study a certain logmodular algebra A on a compact space X and its maximal ideal space $M(A)$ in connection with an abstract Hardy algebra H^∞ associated with A.

§ 1. Introduction.

We denote by B a complex commutative Banach algebra with a unit, and by B^{-1} the group of invertible elements in B. We denote by $M(B)$ the maximal ideal space of B. We denote by $\hat{f}$ the Gelfand transform of $f \in B$, by $\hat{B}$ the set $\{\hat{f} : f \in B\}$, and by $\Gamma(B)$ the Shilov boundary of B. We often write f for $\hat{f}$, since the meaning will be clear from the context.

In § 3 and § 4, we denote by A a weak-* Dirichlet algebra on a nontrivial probability measure space $(X, \mathscr{A}, m)$, and by $H^\infty = H^\infty(m)$ the weak-* closure of A in $L^\infty(m)$. We will often denote by m the complex homomorphism of H^∞ which is determined by the measure m. Let J^∞ be the weak-* closed linear span of all functions in H^∞, each of which vanishes on some set of positive measure. Then J^∞ is an ideal of H^∞ which is contained in $H_m^\infty = \{f \in H^\infty : \int f\, dm = 0\}$. In [14], we call J^∞ the typical ideal. In [14], we have established a decomposition $H^\infty = \mathscr{H}^\infty \oplus I^\infty$ with I^∞, a spacific ideal of H^∞ with $I^\infty \subset J^\infty$, where $\oplus$ denotes the algebraic direct sum (see § 2). Let $\mathscr{L}^\infty$ (resp. N^∞) be the weak-* closure of $\mathscr{H}^\infty + \overline{\mathscr{H}^\infty}$ (resp. $I^\infty + \overline{I^\infty}$) (the bar denotes conjugation). Let $\tilde{X} = M(L^\infty(m))$, $Y = \Gamma(H^\infty | \text{hull } I^\infty)$ and let $E(I^\infty)$ be the support set of I^∞. For $\phi \in Y$, let $\mathscr{K}(\phi) = \{\tilde{x} \in \tilde{X} : f(\tilde{x}) = \phi(f)\ \forall f \in \mathscr{L}^\infty\}$. For any measurable set E of X, χ_E denotes the characteristic function of E. For any set E of a topological space X, $\bar{E}$ denotes the closure of E in X.

In § 3, we obtain the following. (i) $\phi \in \text{hull } I^\infty$ belongs to Y if and only if $|\phi(f)| = 1$ for every inner function f in $\mathscr{H}^\infty$. (ii) Theorem 3.5.