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Nevanlinna and Smirnov classes on the upper half plane

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1. Introduction and notations

The Nevanlinna and Smirnov classes defined on the unit disk U in $\boldsymbol{C}$ will be denoted by $N(U)$ and $N_*(U)$, respectively. In this paper, we shall define the Nevanlinna class, $N_0(D)$, and the Smirnov class, $N_*(D)$, on $D:=\{z\in\boldsymbol{C}|\mathrm{Im}\, z>0\}$. Yanagihara and Nakamura [10, 8. (3)] posed a problem to introduce the Smirnov class on D; our treatment will be an answer. We let $N_0(D)$ consist of all holomorphic functions f on D such that

$$d(f,0):=\sup_{y>0}\int_{\boldsymbol{R}}\log\ (1+|f(x+iy)|)dx<+\infty,$$

and we let $N_*(D)$ consist of f such that $\log\ (1+|f(z)|)\leqq P[\phi](z)\ (z\in D)$ for some $\phi\in L^1(\boldsymbol{R})$, $\phi\geqq 0$, where the right side means the Poisson integral. $N_0(D)$ is an algebra over $\boldsymbol{C}$ and $N_*(D)$ is its subalgebra. First we prove a factorization theorem for functions in $N_0(D)$, as Krylov [4] does for functions in the class $\mathfrak{N}$. $\mathfrak{N}$ is defined by L^1-boundedness of $\log^+|f(x+iy)|$ and, since $1\in\mathfrak{N}$ and $2\notin\mathfrak{N}$, this is not a vector space. $N(U)$ and $N_*(U)$ have remarkable topological properties, as shown by Shapiro and Shields [6] and Roberts [5]. We shall show that our classes have very similar properties. On the other hand, it will be proved that $N_0(D)$ and $N_*(D)$ cannot be linearly isometric to $N(U)$ and $N_*(U)$, respectively, in contrast to the fact that $H^p(D)$ are linearly isometric to $H^p(U)$ for all p, $0<p\leqq+\infty$.

We denote by σ the normalized Lebesgue measure on T, the unit circle in $\boldsymbol{C}$. Let $\Psi(z)=(z-i)(z+i)^{-1}\ (z\in\bar{D})$. Let ν be a real measure on T. Then there corresponds a finite real measure μ on $\boldsymbol{R}$ such that

$$\int_{\boldsymbol{R}}h(t)d\mu(t)=\int_{T^*}(h\circ\Psi^{-1})(\eta)d\nu(\eta)\quad(h\in C_c(\boldsymbol{R})),$$

where $T^*=T\setminus\{1\}$. Denoting the kernel $(\eta+w)(\eta-w)^{-1}$ by $H(w,\eta)$ $((w,\eta)\in U\times T)$, we can write