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An existence theorem of foliations with singularities A_k, D_k and E_k

Dedicated to Professor Masahisa Adachi on his 60th birthday

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§ 1. Introduction

In this paper a smooth (C^∞) singular foliation $\mathscr{H}$ of codimension q on a smooth manifold N of dimension n means an equivalence class of an open covering $\{V_s\}_{s\in I}$ of N and a family of smooth maps $\phi_s: V_s\to \boldsymbol{R}^q$ and C^∞-diffeomorphisms $h_{st}(x)$ for s, $t\in I$ and each $x\in V_s\cap V_t$ satisfying cocycle conditions (c. f. [9]). Recall the singularities A_k, D_k, and E_k of smooth functions in [4] and denote one of them by X_k for simplicity. It is known that there exist the submanifolds ΣX_k in $J^\infty(N, \boldsymbol{R}^q)$ such that a smooth map germ $\phi:(N, x)\to(\boldsymbol{R}^q, y)$ is C^∞ equivalent to a C^∞ stable unfolding of a smooth function germ with singularity of type X_k at the corresponding point if and only if the infinite jet map $j\phi: N\to J^\infty(N, \boldsymbol{R}^q)$ is transverse to ΣX_k and $j\phi(x)\in\Sigma X_k$. ΣA_k is the well known Boardman manifold $\Sigma^{n-q+1,1,\cdots,1,0}$ in [5] and see the difinition of ΣD_k and ΣE_k in [3]. So we say in this paper that a point x of N is a singular point of type X_k of $\mathscr{H}$ if $x\in V_s$ for some $s\in I$ and $j\phi_s(x)$ belongs to ΣX_k in $J^\infty(V_s, \boldsymbol{R}^q)$.

The purpose of this paper is to reduce an existence problem of a smooth singular foliation having a class of given singularities of type A_k, D_k and E_k to a homotopy-theoretic one. The result will be stated in a formulation motivated by [7] and [11].

Let P be another smooth manifold of dimension p with smooth (non-singular) foliation $\mathscr{F}$ of codimension q represented by a covering $\{U_i\}_{i\in J}$ of P and a family of smooth maps $\psi_i: U_i\to\boldsymbol{R}^q$. We define the submanifold $\Sigma X_k(\mathscr{F})$ in $J^\infty(N, P)$ as follows. Let $j(\psi_i): J^\infty(N, U_i)\to J^\infty(N, R^q)$ be the induced submersion of ψ_i mapping a jet $z=jf(x)$ onto $j(\psi_i\circ f)(x)$ and $j(u_i): J^\infty(N, U_i)\to J^\infty(N, P)$ be the induced jet map of the inclusion u_i of U_i into P. Then we set $\Sigma X_k(\mathscr{F})$ is the union of all submanifolds $j(u_i)(j(\psi_i)^{-1}(\Sigma X_k))$ for all $i\in J$. It does not depend on the choice of $\{U_i, \psi_i\}$. Let $\Omega(\mathscr{F})$ be any open subbundle of $J^\infty(N, P)$ consisting of a number of (possibly infinite) submanifolds $\Sigma X_k(\mathscr{F})$ and of all