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Transversely piecewise linear foliation by planes and cylinders ; PL version of a theorem of E. Ghys

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§ 0. Introduction

Let Σ be a closed oriented surface of genus ≥ 2 and $p: E\rightarrow\Sigma$ a oriented S^{1}-bundle over Σ. Then there exists the cohomology class $e(E)\in H^{2}(\Sigma, \boldsymbol{Z})$ which is known as the Euler class. We define the Eular number $eu(E)\in\boldsymbol{Z}$ by the formula

$$eu(E)=\langle e(\mathrm{E}), [\Sigma]\rangle.$$

Here $[\Sigma]\in H_{2}(\Sigma, \boldsymbol{Z})$ denotes the fundamental class of Σ.

The S^{1}-bundle E has a codimension-one foliation $\mathscr{F}$ transverse to each fiber of it if and only if it satisfies the inequality

$$|eu(E)|\leq|\chi(\Sigma)|,$$

where $\chi(\Sigma)$ denotes the Euler characteristic of Σ (see [Mil], [Wo]).

Recently E. Ghys found the influence of the qualitative properties of $\mathscr{F}$ on the Euler number $eu(E)$ of E in his paper [Gh]. In order to see this, we will explain the minimal set of $\mathscr{F}$, and the classification of it first.

Let M be a closed manifold and $\mathscr{G}$ a codimension-one foliation of M. A subset S of M is *saturated* if it is a union of leaves of $\mathscr{G}$. Non-empty, closed, saturated subset $\mathscr{M}$ of M is called *minimal* if it is minimal about these properties. Since M is compact, there exists a minimal set $\mathscr{M}$ of M. Any minimal set $\mathscr{M}$ is one of the following three types:

(1) a closed leaf,
(2) M (in this case, the foliation $\mathscr{G}$ is called *minimal*),
(3) an exceptional minimal set (that is, for any point $x\in\mathscr{M}$, there exists a compact arc T through x in M such that $\mathscr{M}\cap T$ is a Cantor set).

Now let $E, \mathscr{F}, \Sigma$ be as above . As is well known, if $\mathscr{F}$ has a closed leaf, then $eu(E)=0$. And for any integer n with $|n|\leq|\chi(\Sigma)|$, there exists a transversely projective foliated S^{1}-bundle $(E_{n}, \mathscr{F}_{n})$ over Σ such that $eu(E_{n})=n$. The result of E. Ghys ([Gh]) mentioned above is as fol-