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Geometric structures on filtered manifolds

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Dedicated to Professor Noboru Tanaka
on the occasion of his sixtieth birthday

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Introduction

Geometry deals with spaces and structures. In differential geometry, the spaces are usually differential manifolds and the structures are usually defined on them in terms of differential quantities and called geometric structures of order k if the defining quantities involve only derivatives of order up to k. The general equivalence problem is to find criteria to decide whether or not two geometric structures are (locally) equivalent. It is to this problem that the present work is devoted.

Let us first briefly mention the background. The general equivalence problem has been studied by many geometers since S. Lie. In particular, É. Cartan, in his study of infinite groups [1], invented a general method to treat the equivalence problem on the basis of the method of moving frames and the theory of Pfaff systems in involution, and found important applications in various domains of his work. However, his method was rather of the nature of a general heuristic principle not settled in precise mathematical concepts.

As was brought to light by C. Ehresmann and others, one of the fundamental concepts underlying his method is that of principal fibre bundle and G-structure. The extensive works which followed, in particular, I. M. Singer - S. Sternberg [21] and S. Sternberg [22], gave a rigorous foundation to deal with the general equivalence problem as that of G-structures and clarified important aspects of Cartan's ideas.

But the theory of G-structures as achieved there did not seem adequate to treat the equivalence problem in full generality: Even if one confines oneself to the equivalence problem of G-structures (the first order geometric structures), one has to deal with higher order geometric structures in a way suitable to find the higher order invariants of G-structures, and moreover it is necessary to develop a theory including the intransitive stuctures.