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A unit group in a character ring of an alternating group II

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1. Introduction

Throughout this paper, G denotes always a finite group, Z the ring of rational integers, Q the field of rational numbers, C the field of complex numbers. In addition, we fix the following notations.

$R(G)$; a character ring of G
$U(R(G))$; a unit group of $R(G)$
$U_f(R(G))$; the subgroup of $U(R(G))$ which consists of units of finite order in $R(G)$
S_n, A_n ; a symmetric group and an alternating group on n symbols respectively for a natural number n.

In the paper of [**6**], we proved the following theorem.

THEOREM 1.1. *rank* $U(R(A_n))/\{\pm 1\}=c(n)$.
(*See Definition* 2.3 *concerning a number* $c(n)$)

In section 3, we will construct $c(n)$ units $\psi_1, \ldots, \psi_{c(n)}$ in $R(A_n)$ and show that $U^2(R(A_n))\subseteqq\langle\psi_1, \ldots, \psi_{c(n)}\rangle$, where $U^2(R(A_n))=\{\psi^2|\psi\in U(R(A_n))\}$ and $\langle\psi_1, \ldots, \psi_{c(n)}\rangle$ is an abelian subgroup of $U(R(A_n))$ generated by $\psi_1, \ldots, \psi_{c(n)}$. (See Theorem 3.4.). It is easily proved that rank $\langle\psi_1, \ldots, \psi_{c(n)}\rangle=c(n)$. (See the proof of Lemma 4.1 of [**6**]), and so Theorem 1.1 is a direct consequence of the above result.

For a given unit ψ in $R(A_n)$, we will give the necessary and sufficient condition on which ψ is the difference of two irreducible C-characters of A_n. (See Theorem 3.6.)

In section 4, as an application of the above results, we will state some examples such that the equation $\{\pm 1\}\times\langle\psi_1, \ldots, \psi_{c(n)}\rangle=U(R(A_n))$ holds, by the way of finding generators of $U(R(A_n))$ concretely, and we will also give the example such that a unit in $R(A_n)$ is the difference of two irreducible C-characters of A_n.

Now we pay attention to the fact that for $n=3$, 4, $U(R(A_n))=U_f(R(A_n))=\{\pm\chi_1, \pm\chi_2, \pm\chi_3\}$, where χ_1, χ_2, and χ_3 are the linear characters of