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Radon transform of hyperfunctions and support theorem

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Abstract. We define the Radon transform for a class of hyperfunctions which are not necessarily with bounded support. We give characterization of the image space for some basic spaces. Then we give a variant of support theorem by Helgason-Boman.

In this article we define the Radon transform for a class of hyperfunctions and discuss its properties. Especially, we prove a variant of support theorem by Helgason-Boman. Although the significance of extending the Radon transform to hyperfunctions is not so clear from the viewpoint of applications to industrial tomography, it will be interesting from purely mathematical viewpoint. Here we only treat the codimension one case i.e. the case of hyperplane integrals.

We should remark that in the theory of hyperfunctions there already exists another kind of Radon transformation theory (see e.g. [Kt]). Its viewpoint lies in the microlocalization of the classical Radon transformation and is different from ours laying stress on the global behavior of the transformation.

1. Introduction. Hyperfunctions

In this section we first give a short review on (Fourier) hyperfunctions, and then discuss the possibility of their Radon transform. For further details on (Fourier) hyperfunctions see [Kn2] and references therein.

A hyperfunction $f(x)$ on $\boldsymbol{R}^n$ is the equivalence class of formal expressions of the form

$$f(x)=\sum_{j=1}^{N} F_j(x+i\Gamma_j 0), \tag{1.1}$$

where Γ_j denotes an open convex cone with vertex at the origin and $F_j(z)$ a function holomorphic in a wedge-like domain with asymptotic form $\boldsymbol{R}^n+i\Gamma_j$ at the real axis. The equivalence means the natural rewriting among the defining functions F_j, and its precise expression is given by Martineau's edge of the wedge theorem which is a concrete expression of