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Cohen-Macaulay types of Hall lattices

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Abstract. The submodule lattice of a finite modules over a discrete valuation ring is called the Hall lattice. In this paper, extending the previous work [7], we consider Cohen-Macaulay types of Hall lattices and show that they are polynomials in q (q is the number of elements of the residue field of the discrete valuation ring) with integer coefficients.

Key words: Stanley-Reisner rings, Cohen-Macaulay posets, Möbius functions, partitions, Hall polynomials, Gaussian polynomials.

1. Introduction

In this section, we summarize basic definitions and results about partially ordered sets (posets, for short) and Stanley-Reisner rings. We begin with the definition of Stanley-Reisner rings of finite posets and Cohen-Macaulay types of them. See [2], [3], [8] for precise informations.

Let P be a poset. In this paper, the cardinality $\sharp P$ of a poset P is always finite. We consider a polynomial ring $A = K[x \mid x \in P]$ over a field K whose indeterminates are the elements of P. Let I be the ideal of A generated by the set of all the monomials $\{xy \in A \mid x \in P \text{ and } y \in P \text{ are incomparable.}\}$. The quotient ring A/I is called the *Stanley-Reisner ring* of P over K. A *finite free resolution* of $K[P]$ over A is an exact sequence of A-modules

$$0 \longrightarrow F_h \longrightarrow \cdots \longrightarrow F_1 \longrightarrow F_0 \longrightarrow K[P] \longrightarrow 0,$$

where each F_i is a free A-module of finite rank r_i. Here we can minimize h and all r_i's simultaneously [2]. The minimal one is called the *minimal free resolution* of $K[P]$ over A. The minimal free resolution always exists and is uniquely determined. Minimal free resolutions are one of a main interest in the commutative ring theory for a reason that we can compute the Hilbert function of a ring from that [2, p.151]. If the above sequence is a minimal free resolution of $K[P]$ over A, then $\beta_i = \operatorname{rank} F_i$ is called the i-th *Betti number* and $h = \operatorname{hd}_A(K[P])$ the *homological dimension* of $K[P]$ over A.

The homological dimension $\operatorname{hd}_A(K[P])$ is estimated as follows. Let v be

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