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Separation and weak separation on Riemann surfaces

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Abstract. We show some necessary and sufficient conditions for weak separation by an algebra A of analytic functions on a Riemann surface R. One of these equivalent conditions is the following. There exists a sequence of relatively compact open sets $\{D_n\}$ in R such that (i) ∂D_n is connected, (ii) $\overline{D}_1 \subset \overline{D}_2 \subset \overline{D}_3 \subset \cdots$, (iii) $R = \cup \overline{D}_n$, and (iv) A separates the points of a neighborhood of ∂D_n.

Key words: weak separation, algebra of analytic functions, Riemann surface.

1. Introduction

Let R be a Riemann surface, and let A be an algebra of analytic functions on R. We always assume that A contains constant functions. We say that points p and q of R are separated by A if there is a function f in A such that $f(p) \neq f(q)$, and when any pair of distinct points are separated by A, we say that the algebra A separates the points of R. For functions f and g in A such that $g \not\equiv 0$, (f/g) is a meromorphic function and so we can consider the value $(f/g)(p)$ at any point p of R. According to Royden [4] we say that points p and q of R are weakly separated by A if there are functions f and g in A as above such that $(f/g)(p) \neq (f/g)(q)$, and when any pair of distinct points are weakly separated by A, we say that the algebra A weakly separates the points of R.

On the other hand, in Gamelin-Hayashi [2] it was defined that A weakly separates the points of R if there is a discrete subset Λ of R such that A separates the points of $R \backslash \Lambda$ in case A is the algebra of bounded analytic functions $H^\infty(R)$. These two definitions for weak separation coincides each other.

In this paper we study some necessary and sufficient conditions for weak separation, and show that separation on a rather narrow set means weak separation on R. We also include a proof of equivalence of two definitions for weak separation. It will be convenient since the proof is not given in [2]. For the moment we use the terminology "weak separation" in the sense of

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