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On the number of singularities of a generic surface with boundary in a 3-manifold

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Abstract. We consider a C^∞ generic map $f : M \to N$ of a compact surface M with boundary into a 3-manifold N with boundary which is neat (i.e., $f^{-1}(\partial N) = \partial M$). The isolated singularities of the image $f(M)$ are triple points, cross caps and boundary double points. Under certain homological conditions, we give some formulae relating the numbers of these singularities. We also obtain some geometrical applications of these results.

Key words: singular surface, triple point, cross cap, boundary double point, selftranslation surface.

1. Introduction

It is well known that a C^∞ generic map $f : M \to N$ of a closed surface M into a 3-manifold N is an immersion with normal crossings except at isolated points, at which f has cross caps (for example, see [9]). Thus the singular part of $f(M)$ is made by the curve of double points of f with intersections at the triple points and with end points at the cross caps. In particular, there are a finite number of triple points and cross caps. In [18], Szücs gives the following congruence relating the number of triple points to the number of cross caps.

Theorem 1.1 *Let $f : M \to \mathbf{R}^3$ be a C^∞ generic map of a closed surface M into $\mathbf{R}^3$. Then*

$$T(f) \equiv \sum_{i=1}^{k} n(x_i, f) + \chi(M) \mod 2,$$

where $T(f)$ is the number of triple points of f, $\chi(M)$ is the Euler characteristic of M, $x_1, \ldots, x_k \in f(M)$ are the cross caps of f, and $n(x_i, f)(\in \{0, 1\})$ is the index of the cross cap x_i conveniently defined.

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