Hokkaido Mathematical Journal Vol. 27 (1998) p. 505–515

Higher Specht polynomials for the complex reflection group $G(r, p, n)$

(To Professor Takeshi Hirai on his sixtieth birthday)

H. MORITA and H.-F. YAMADA

(Received December 16, 1996; Revised December 12, 1997)

Abstract. A basis of the quotient ring P/J_+ is given, where P is the ring of polynomials and J_+ is the ideal generated by the fundamental invariants under the action of the complex reflection group $G(r, p, n)$.

Key words: complex reflection groups, coinvariant rings, Clifford theory, tableaux.

1. Introduction

This note is concerned with a certain graded module over the imprimitive complex reflection group $G(r, p, n)$ [ST]. The group $G(r, p, n)$ ($r, p, n \geq 1$, $p|r$) consists of the monomial matrices whose nonzero entries are of the form ζ^j ($0 \leq j < r$) and such that the d-th power of the product of all nonzero entries is equal to 1, where we denote by ζ a primitive r-th root of 1, and $d = r/p$. In some special cases, $G(r, p, n)$ is isomorphic to the Weyl group:

$$\begin{aligned} G(1,1,n) &= W(A_{n-1}), \\ G(2,1,n) &= W(B_n) = W(C_n), \\ G(2,2,n) &= W(D_n), \\ G(6,6,2) &= W(G_2). \end{aligned}$$

Also it is naturally identified as a normal subgroup of the wreath product

$$G(r,n) = (\mathbf{Z}/r\mathbf{Z}) \wr S_n = \{(\zeta^{i_1}, \ldots, \zeta^{i_n}; \sigma) \mid i_k \in \mathbf{N},\ \sigma \in S_n\},$$

whose product is given by

$$(\zeta^{i_1}, \ldots, \zeta^{i_n}; \sigma)(\zeta^{j_1}, \ldots, \zeta^{j_n}; \tau) = (\zeta^{i_1 + j_{\sigma^{-1}(1)}}, \ldots, \zeta^{i_n + j_{\sigma^{-1}(n)}}; \sigma\tau).$$

1991 Mathematics Subject Classification : 05E10, 20C05.