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A periodic boundary value problem for a generalized 2D Ginzburg-Landau equation

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Abstract. This article studies the periodic boundary value problem for a generalized Ginzburg-Landau equation with additional fifth order term and cubic terms containing spatial derivatives. We present sufficient condition for global existence. A blow-up of solutions is found via numerical simulation.

Key words: generalized Ginzburg-Landau equation, global solution, blow-up.

1. Introduction

The classical one-dimensional Ginzburg-Landau equation (GL)

$$u_t = (\nu + i\alpha)u_{xx} - (\kappa + i\beta)|u|^2 u + \gamma u \tag{1-1}$$

frequently occurs as the leading term in an asymptotic expansion of the slowly varying envelope of solutions for such "exact" models such as the Navier-Stocks equations [1]. If $\kappa < 0$ then as γ increases, (1-1) with periodical boundary condition undergoes a subcritical bifurcation after which almost all solutions become unbounded in finite time. It is also of physical interest (see [2–4] for details) to carry the expansion to second order in case of small κ. This leads to the resulting generalized GL [5].

$$u_t = \alpha_0 u + \alpha_1 u_{xx} + \alpha_2 |u|^2 u + \alpha_3 |u|^2 u_x + \alpha_4 u^2 \bar{u}_x + \alpha_5 |u|^4 u \tag{1-2}$$

where $\alpha_j = a_j + ib_j$ are all complex parameters (though we note that α_0 can be regarded as real since the complex part can be eliminated via a simple transformation). If $a_1 > 0 > a_5$ and $-4a_1a_5 > (b_3 - b_4)^2$ then (1-2) possesses a global classical solution $u(t) \in C([0,\infty); H^1_{per}[0,L]) \cap C^1((0,\infty); H^1_{per}[0,L])$ for every $u(0) \in H^1_{per}[0,L]$ [2]. It has been found that the cubic terms involving partial derivatives can significantly slow the

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