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Perturbations of Weyl-Heisenberg frames

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Abstract. We develop a usable perturbation theory for Weyl-Heisenberg frames. In particular, we prove that if $(E_{mb}T_{na}g)_{m,n\in\mathbb{Z}}$ is a WH-frame and h is a function which is close to g in the Wiener Amalgam space norm, then also $(E_{mb}T_{na}h)_{m,n\in\mathbb{Z}}$ is a WH-frame. We also prove perturbation results for the parameters a, b.

Key words: Weyl-Heisenberg frame, Gabor frame, perturbation, Riesz basis, frame.

1. Introduction

In 1952, Duffin and Schaeffer [10] introduced the notion of a frame for a Hilbert space. A sequence $(f_i)_{i\in I}$ is a **frame** for a Hilbert space H if there are constants $A, B > 0$ satisfying,

$$A\|f\|^2 \le \sum_{i\in I} |\langle f, f_i\rangle|^2 \le B\|f\|^2, \tag{1.1}$$

for all $f \in H$. The constant A (respectively, B) is a **lower** (resp. **upper**) frame bound for the frame. A frame (f_i) can be considered as a "generalized basis": using the fact that the **frame operator** $Sf = \sum\langle f, f_i\rangle f_i$ is a bounded invertible operator on H, every $f \in H$ can be represented as a convergent series

$$f = SS^{-1}f = \sum\langle f, S^{-1}f_i\rangle f_i.$$

$(f_n)_{n\in I}$ is a **Riesz basic sequence** if there exist constants $A, B > 0$ such that

$$A\sum_{i\in I}|c_i|^2 \le \left\|\sum_{i\in I} c_i f_i\right\|^2 \le B\sum_{i\in I}|c_i|^2, \tag{1.2}$$

for all finite sequences $\{c_i\}_{i\in I}$. If also $\overline{\text{span}}(f_n)_{n\in I} = H$, then $(f_n)_{n\in I}$ is a **Riesz basis**. Alternatively, a Riesz basis is a frame which is at the

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