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Fully discrete approximation by Galerkin Runge-Kutta methods for quasilinear parabolic systems

(Dedicated to Professor Norio Shimakura on the occasion of his sixtieth birthday)

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Abstract. We study fully discrete approximation of quasilinear parabolic systems. Presenting a full discretization scheme based on the Galerkin and the Runge-Kutta methods, we establish the stability and the error estimate of the scheme by means of the semigroup method. First our results are stated for a chemotaxis-growth system arising in biology, then those are generalized to quasilinear abstract parabolic evolution equations.

Key words: quasilinear parabolic systems, implicit Runge-Kutta methods, Galerkin finite element method.

1. Introduction

This paper is concerned with a numerical analysis for a quasilinear diffusion system

$$\text{(CG)} \qquad \begin{cases} \dfrac{\partial u}{\partial t} = a\triangle u - \nabla\cdot\{u\nabla B(\rho)\} + c(u) & \text{in } \Omega\times(0,\infty), \\ \dfrac{\partial \rho}{\partial t} = d\triangle\rho + fu - g\rho & \text{in } \Omega\times(0,\infty), \\ \dfrac{\partial u}{\partial n} = \dfrac{\partial \rho}{\partial n} = 0 & \text{on } \partial\Omega\times(0,\infty), \\ u(x,0) = u_0(x),\ \rho(x,0) = \rho_0(x) & \text{in } \Omega. \end{cases}$$

This system was presented by Mimura and Tsujikawa [14] as a mathematical model describing aggregating patterns induced by the effects of chemotaxis and growth. $u(x,t)$ and $\rho(x,t)$ denote the population density of biological individuals and the concentration of chemical substance, respectively, at a

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