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On the class of univalent functions starlike with respect to N-symmetric points

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Abstract. In the present paper we study certain generalizations of the class $\mathcal{SSP}_N$ of functions starlike with respect to N-symmetric points. We obtain a structural formula for functions in $\mathcal{SSP}_N$, and deduce a sharp lower bound for $|f'(z)|$ when N is even (this case completes the distortion theorem for $\mathcal{SSP}_N$). Improved estimates for Koebe constants are also given. Further, it is proved that for any $N \geq 2$ the class $\mathcal{SSP}_N$ contains non-starlike functions. Finally, we characterize the class $\mathcal{SSP}_N$ in terms of Hadamard convolution.

Key words: univalent, starlike, close-to-convex and convex functions.

1. Introduction and main results

Denote by $\mathcal{A}$ the class of all functions f, analytic in the unit disc Δ and normalized by $f(0) = f'(0) - 1 = 0$. Let $\mathcal{S}$ be the class of functions in $\mathcal{A}$ that are univalent in Δ. A function $f \in \mathcal{A}$ is said to be starlike with respect to symmetric points [8] if for any r close to 1, $r < 1$, and any z_0 on the circle $|z| = r$, the angular velocity of $f(z)$ about the point $f(-z_0)$ is positive at z_0 as z traverses the circle $|z| = r$ in the positive direction, i.e.,

$$\operatorname{Re}\left(\frac{zf'(z)}{f(z) - f(-z_0)}\right) > 0, \quad \text{for } z = z_0, \quad |z| = r.$$

Denote by $\mathcal{SSP}$ the class of all functions in $\mathcal{S}$ which are starlike with respect to symmetric points and, functions f in this class is characterized by

$$\operatorname{Re}\left(\frac{zf'(z)}{f(z) - f(-z)}\right) > 0, \quad z \in \Delta.$$

We also have the following generalization of the class $\mathcal{SSP}$ introduced by K. Sakaguchi [8]. For $f(z) = z + \sum_{k=2}^{\infty} a_k z^k \in \mathcal{A}$, set

$$\mathcal{SSP}_N = \left\{ f \in S : \operatorname{Re}\left(\frac{zf'(z)}{f_N(z)}\right) > 0, \quad z \in \Delta \right\},$$

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