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Singular limit of a second order nonlocal parabolic equation of conservative type arising in the micro-phase separation of diblock copolymers

(Dedicated to Professors Masayasu Mimura and Takaaki Nishida on their sixtieth birthday)

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Abstract. We study the limiting behavior as ε tends to zero of the solution of a second order nonlocal parabolic equation of conservative type which models the micro-phase separation of diblock copolymers. We consider the case of spherical symmetry and prove that as the reaction coefficient tends to infinity the problem converges to a free boundary problem where the interface motion is partly induced by its mean curvature.

Key words: reaction-diffusion systems of conservative type, singular limits, nonlocal motion by mean curvature, asymptotic expansions.

1. Introduction

In this paper, we consider a second order nonlocal parabolic equation of conservative type proposed by Ohnishi and Nishiura [9], namely

$$
(\mathcal{P}^\varepsilon)\begin{cases}
u_t^\varepsilon = \Delta u^\varepsilon + \dfrac{1}{\varepsilon^2}\Big(f(u^\varepsilon) - \displaystyle⨍_\Omega f(u^\varepsilon) - \varepsilon v^\varepsilon\Big) & \text{in } \Omega\times(0,T) \quad (1.1)\\
-\Delta v^\varepsilon = u^\varepsilon - \displaystyle⨍_\Omega u^\varepsilon & \text{in } \Omega\times(0,T) \quad (1.2)\\
\dfrac{\partial u^\varepsilon}{\partial n} = \dfrac{\partial v^\varepsilon}{\partial n} = 0 & \text{in } \partial\Omega\times(0,T) \quad (1.3)\\
\displaystyle⨍_\Omega v^\varepsilon dx = 0 & \text{for } t\in(0,T) \quad (1.4)\\
u^\varepsilon(x,0) = u_0^\varepsilon(x) & \text{for } x\in\Omega \quad (1.5)
\end{cases}
$$

where

$$f(s) := 2s(1-s^2), \quad ⨍_\Omega u\,dx := \frac{1}{|\Omega|}\int_\Omega u\,dx$$

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