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Notes on commutators and Morrey spaces

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Abstract. We show that the commutator $[M_b, I_\alpha]$ of the multiplication operator M_b by b and the fractional integral operator I_α is bounded from the Morrey space $L^{p,\lambda}(R^n)$ to the Morrey space $L^{q,\lambda}(R^n)$ where $1 < p < \infty$, $0 < \alpha < n$, $0 < \lambda < n - \alpha p$ and $1/q = 1/p - \alpha/(n - \lambda)$ if and only if b belongs to $BMO(R^n)$.

Key words: commutator, fractional integral, Morrey space.

1. Introduction

Let I_α, $0 < \alpha < n$, be the fractional integral operator defined by

$$I_\alpha f(x) = \int_{R^n} \frac{f(y)}{|x-y|^{n-\alpha}}\, dy.$$

We consider the commutator

$$[M_b, I_\alpha]f(x) = b(x) I_\alpha f(x) - I_\alpha(bf)(x), \qquad b \in L^1_{\mathrm{loc}}(R^n).$$

Chanillo [1] and the first author [7] obtained the necessary and sufficient condition for which the commutator $[M_b, I_\alpha]$ is bounded on $L^p(R^n)$. Di Fazio and Ragusa [4] obtained the necessary and sufficient condition for which the commutator $[M_b, I_\alpha]$ is bounded on Morrey spaces for some α.

In this paper we refine their results in [4] by using the duality argument and the factorization theorem for $H^1(R^n)$ (Theorem 2). Our proof is different from the one in [4].

2. Definitions and Notations

For a set $E \subset R^n$ we denote the characteristic function of E by χ_E and $|E|$ is the Lebesgue measure of E.

We denote a ball of radius t centered at x by $B(x,t) = \{y; |x-y| < t\}$.

Definition 1 Let $1 \le p < \infty$, $\lambda \ge 0$. We define the classical Morrey space by

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