Global stability in discrete models of nonautonomous Lotka-Volterra type

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Abstract. In this paper, we establish sufficient conditions for the global asymptotic stability of the following discrete models of nonautonomous Lotka-Volterra type:

$$\begin{cases} N_i(p+1) = N_i(p) \exp\Bigl\{c_i(p) - a_i(p) N_i(p) - \sum_{j=1}^n \sum_{l=0}^m a_{ij}^l(p) N_j(p-k_l)\Bigr\}, \\ 1 \leq i \leq n, & \text{for } p = 0, 1, 2, \dots, \\ N_i(p) = N_{ip} \geq 0, & \text{for } p \leq 0, & \text{and } N_{i0} > 0, \ 1 \leq i \leq n, \end{cases}$$
 where each $c_i(p), a_i(p)$ and $a_{ij}^l(p)$ are bounded for $p \geq 0$ and
$$\begin{cases} & \inf_{p \geq 0} a_i(p) > 0, \ a_{ii}^0(p) \equiv 0, \ a_{ij}^l(p) \geq 0, \ 1 \leq i \leq j \leq n, \ 0 \leq l \leq m, \\ k_0 = 0, & \text{integers } k_l \geq 0, \ 1 \leq l \leq m. \end{cases}$$

$$\left\{ \begin{array}{l} \inf_{p \geq 0} a_i(p) > 0, \ a_{ii}^0(p) \equiv 0, \ a_{ij}^l(p) \geq 0, \ 1 \leq i \leq j \leq n, \ 0 \leq l \leq m, \\ k_0 = 0, \ \text{integers} \ k_l \geq 0, \ 1 \leq l \leq m. \end{array} \right.$$

We establish a condition for the permanence of system and applying the former work (2002, J. Math. Anal. Appl. 273 492-511) to this system, we improve a known condition for the global asymptotic stability of system.

Key words: permanence, global asymptotic stability, discrete model of nonautonomous Lotka-Volterra type.

Introduction 1.

Consider the following discrete system of nonautonomous Lotka-Volterra type:

$$\begin{cases} N_{i}(p+1) = N_{i}(p) \exp\left\{c_{i}(p) - a_{i}(p)N_{i}(p) - \sum_{j=1}^{n} \sum_{l=0}^{m} a_{ij}^{l}(p)N_{j}(p-k_{l})\right\}, \\ 1 \leq i \leq n, & \text{for } p = 0, 1, 2, \dots, \\ N_{i}(p) = N_{ip} \geq 0, & \text{for } p \leq 0, \text{ and } N_{i0} > 0, 1 \leq i \leq n, \end{cases}$$

$$(1.1)$$

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