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Ergodic theorems for dynamical semi-groups on operator algebras

By Seiji WATANABE

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1. Introduction

Recently the theory of quantum dynamical semi-groups have made very interesting progress ([2, 3, 7, 13]). In the present paper we shall show various ergodic theorems (that is, a mean ergodic theorem, an individual ergodic theorem and a local ergodic theorem) for such dynamical semi-groups. As is well known, von Neumann's mean ergodic theorem was generalized to many abstract spaces.

In 1966, I. Kovägcs and J. Szücs [9] proved an ergodic type theorem which is considered as a non-commutative mean ergodic theorem and has many applications in mathematical physics. The individual ergodic theorem was also investigated by many authors, but the almost everywhere convergence can not be discussed on abstract spaces. However, recently E. C. Lance [12] and Y. G. Sinai and V. V. Anshelvich [17] have shown non-commutative analogues of the Birkhoff's individual ergodic theorem for automorphisms of operator algebras. As is stated in [12], [17], we can discuss "almost everywhere convergence" on von Neumann algebras because of its rich structure as the natural non-commutative generalization of L^{∞}-algebras over probability measure spaces. There is no notion of underlying measure space in the present situation. However, Egoroff's theorem implies that a sequence of measurable functions converges almost everywhere if and only if the sequence converges uniformly on measurable subsets of measure arbitrary close to 1. The uniform convergence for bounded functions corresponds to the norm convergence in L^{∞}-algebras and measurable subsets correspond to its characteristic functions. Thus we may restate the almost everywhere convergence by means of the measure, L^{∞}-norm and the characteristic functions. This restatements fit naturally into our setting. On the other hand, the so-called local ergodic theorem, which was first established by N. Wiener, gives a powerful tool to investigate the asymptotic behavior of dynamical systems as the time parameter $t\rightarrow+0$. This result was also extended to the general settings by many authors. We present