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On almost Blaschke manifolds II

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§ 0. Introduction

In a previous paper [5] the auther studied the topology of a compact riemannian manifold (M, g) whose injectivity radius $i(M)$ is close to its diameter $d(M)$ and got the following results.

THEOREM A. *Let (M, g) be a 2-dimensional riemannian manifold and K denote its Gaussian curvature. Assume that one of the following holds;*
(i) *There is a positive number δ such that*

$$K \geq -\delta^2 \text{ and } \sinh(\delta i(M)) > (\sqrt{3}/2)\sinh(\delta d(M)).$$

(ii) *$K \geq 0$ and $i(M) > (\sqrt{3}/2)d(M)$ hold.*
Then M is diffeomorphic to the sphere S^2 or projective plane P^2.

THEOREM B. *Let (M, g) be a 3-dimensional riemannian manifold and K denote its sectional curvature. If there is a positive number δ such that*

$$K \geq \delta^2 \text{ and } \sin(\delta i(M)) > (\sqrt{3}/2)\sin(\delta d(M)),$$

then M is diffeomorphic to the sphere S^3 or projective space P^3.

These results are best possible.

On the other hand recently O. Durumeric has shown in [4] that in arbitrary dimension any manifold whose injectivity radius is sufficiently close to its diameter has either the trivial fundamental group or the homotopy type of the real projective space.

In this paper we prove the following theorem.

THEOREM. *Let (M, g) be a 3-dimensional compact riemannian manifold and K denote its sectional curvature. Assume that one of the following holds;*
(i) *There is a positive number δ such that we have*

$$K \geq -\delta^2 \text{ and } \sinh(\delta i(M)) > a(\delta d(M)) \cdot \sinh(\delta d(M))$$

where