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Local solvability of semilinear parabolic equations

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Introduction.

The problem of local in time existence of solutions of semilienar parabolic equations has been studied by a number of authors (e.g. [10, 20, 4, 18, 3]) using several different methods. Friedman [10] made use of the $C^{1+\delta}$ estimates (p. 191 therein), also the Sobolevskii-Tanabe method (cf. [22, 3]) gave a contribution to these studies. However, probably the most important innovation in recent years came from H. Amann (see [3] and references therein), who covered general systems of equations using the idea of an extended "variation of constants" formula. Also in [0] independent studies of this problem are presented. Our approach here is more classical, close in spirit but different in method to that of Friedman [10]. The proofs based on a priori estimates and the iteration technique [2] make it possible to study in compact form the equation (1) with linear boundary conditions of the third type. Our estimates (23), (31) of the life time of solutions seems to be new and to have interesting implications (compare e.g. Lemma 4). The form of equation (1) (with the weak assumption of the local Lipschitz continuity of f) covers most of the recently studied single equations with blowing-up solutions [11, 4, 9] as well as problems with blowing-up derivatives [8], also the formally more complete form of the equation in [10, p. 201] (except that we need the coefficients to be smoother). The technique presented here has been used before by the present author [6, 7] in studies on the long time behaviour of solutions of parabolic problems.

Notation.

The notation of Sobolev spaces is taken from an excellent monograph by R. Adams [1], for the Hölder spaces from Ladyženskaja at al. [13] (except that we use the letter C instead of H in [13] to distinguish the notation of Sobolev space). $|\Omega|$ denotes the Lebesgue measure of Ω, $C := |\Omega|^{1/(2n+2)}$. As throughout this paper the space variable x belongs to a fixed bounded domain $\Omega \subset R^n$, hence all the unspecified integrals are understood to be taken over Ω, also unsepcified sums are taken from 1 to n (the space