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The Cauchy problem in abstract Gevrey spaces for a nonlinear weakly hyperbolic equation of second order

Piero D'ANCONA and Renato MANFRIN
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§ 1. Introduction

We investigate here the existence of local solutions to the following abstract Cauchy problem:

$$u''+A(t)u=f(t,\ u(t)) \tag{1}$$
$$u(0)=u_0,\ u'(0)=u_1 \tag{2}$$

in a Hilbert space H, where $A(t)$ is a nonnegative unbounded operator.

In case $A(t)$ satisfies some strict coercivity assumptions, i. e. when (1) is a strictly hyperbolic equation, the local solvability for Pb. (1), (2) is well known, provided $A(t)$ is Lipschitz continuous in time and f is smooth enough. An extensive theory on this problem, embracing most of the concrete results in Sobolev spaces with optimal regularity assumptions, was given by Kato (see [Ka]; see also [LM]).

On the other hand, when $A(t)\geq 0$ is allowed to be degenerate, i.e. when Eq. (1) is of weakly hyperbolic type, then we need much stronger assumptions in order that (1), (2) be locally solvable. This is evident also for linear equations such as

$$u_{tt}=a(t)u_{xx} \tag{3}$$

which may be not locally solvable in C^{∞} for a suitable nonnegative $a(t)\in C^{\infty}$ (see [CS]).

It is possible to overcome this difficulty by requiring that the data and the coefficients are more regular in space variables. Thus in [CJS], [N] it was proved that the equations

$$u_{tt}=\sum_{i,j}a_{ij}(t,x)u_{x_ix_j}+\sum_{j}b_j(t,x)u_{x_j},\ \sum a_{ij}\xi_i\xi_j\geq 0 \tag{4}$$

are globally sovable in the spaces $\gamma^s(\mathbf{R}^n)$ of Gevrey functions of order s, defined as follows:

$$v(x)\in\gamma^s(\mathbf{R}^n)\Longleftrightarrow\forall K\subset\subset\mathbf{R}^n\ \exists C_K,\ \Lambda_K\geq 0:\ |D^{\alpha}v(x)|\leq C_K\Lambda_K^{|\alpha|}\cdot|\alpha|!^s \quad \text{for } x\in K \tag{5}$$