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Brauer functors and Fisher's inequality for t-designs over a finite field with group action

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Abstract. In [I-Y], we proved a Fisher type inequality for a t-design with group action. In this paper, we prove a q-analogue version of a Fisher type inequality in [I-Y]. Let G be a finite group with a normal p-subgroup P and $\mathfrak{B}\subset\begin{bmatrix}V\\k\end{bmatrix}$ a t-design over $\boldsymbol{F}_q$ on which G acts, where V is a finite dimensional vector space over $\boldsymbol{F}_q$ Assume that $2s\leqq t$ and the prime p does not divide $\lambda_s^0\lambda_s^1\cdots\lambda_s^s$. Then $\left|\begin{bmatrix}V\\s\end{bmatrix}^P/G\right|\leqq|\mathfrak{B}^P/G|$, where $\mathfrak{B}^P$ denotes the set of blocks fixed by all elements of P and $\mathfrak{B}^P/G$ denotes the set of G-orbits in $\mathfrak{B}^P$.

1. Introduction

In [I-Y], we proved a Fisher type inequality for a t-design with group action. Let G be a finite group with a normal p-subgroup P and (X, $\mathfrak{B}$) an ordinary (classical) t-design on which G acts. Assume that $2s\leqq t$ and the prime p does not divide $\lambda_s^0\cdots\lambda_s^s$, where λ_i^j is the number of blocks which contain a fixed i-element subset I of X but disjoint from a j-element subset of $X-I$ for $i+j\leqq t$. Then

$$\left|\binom{X}{s}^P/G\right|\leqq|\mathfrak{B}^P/G|, \tag{1}$$

$$\left|\left(\binom{X}{s}^P\times\binom{X}{s}^P\right)/G\right|\leqq\left|\left(\binom{X}{s}^P\times\mathfrak{B}^P\right)/G\right|\leqq|(\mathfrak{B}^P\times\mathfrak{B}^P)/G|, \tag{2}$$

where $\mathfrak{B}^P$ (resp. $\binom{X}{s}^P$) denotes the set of blocks (resp. s-element subsets) fixed by all elements of P and $\mathfrak{B}^P/G$ (resp. $\binom{X}{s}^P/G$) denotes the set of G-orbits in $\mathfrak{B}^P$ (resp. $\binom{X}{s}^P$). In this paper, we consider a q-analogue version of (1) and (2).

A t-$(v, k, \lambda; q)$ *design*, or t-design over $\boldsymbol{F}_q$, is a nonempty collection $\mathfrak{B}$ of k-dimensional subspaces of a v-dimensional vector space over $\boldsymbol{F}_q$