Reduction of Local Uniformization to the Case of Rank One Valuations for Rings with Zero Divisors

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ABSTRACT. This is a continuation of our previous paper, where it was proved that to obtain local uniformization for valuations centered on local domains, it suffices to prove it for rank one valuations. In this paper, we extend this result to the case of valuations centered on rings that are not necessarily integral domains and may even contain nilpotents.

1. Introduction

For an algebraic variety X over a field k, the problem of resolution of singularities is whether there exists a proper birational morphism $X' \longrightarrow X$ such that X' is regular. The problem of local uniformization can be seen as the local version of resolution of singularities for an algebraic variety. For a valuation ν of $k(X)$ having a center on X, the local uniformization problem asks whether there exists a proper birational morphism $X' \longrightarrow X$ such that the center of ν on X' is regular. This problem was introduced by Zariski in the 1940s as an important step to prove resolution of singularities. Zariski's approach consists in proving first that every valuation having a center on the given algebraic variety admits local uniformization. Then these local solutions have to be glued to obtain a global resolution of all singularities.

Zariski [10] succeeded in proving local uniformization for valuations centered on algebraic varieties over a field of characteristic zero. He used this to prove resolution of singularities for algebraic surfaces and threefolds over a field of characteristic zero (see [11]). Abhyankar [1] proved that local uniformization can be obtained for valuations centered on algebraic surfaces in any characteristic and used this fact to prove resolution of singularities for surfaces (see [2] and [3]). He also proved local uniformization and resolution of singularities for threefolds over fields of characteristic other than 2, 3, and 5 (see [4]). Very recently, Cossart and Piltant [5; 6] proved resolution of singularities (and, in particular, local uniformization) for threefolds over any field of positive characteristic and in the arithmetic case. They proved it using the approach of Zariski. However, the problem of local uniformization remains open for valuations centered on algebraic varieties of dimension greater than three over fields of positive characteristic.