

Constructing Banach Spaces of Vector-Valued Sequences with Special Properties

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ABSTRACT. We develop new techniques to construct infinite-dimensional Banach (or quasi-Banach) spaces formed by vector-valued sequences with special properties. We also investigate when such Banach/quasi-Banach spaces can be constructed with the maximum possible dimension. Numerous concrete applications of the results we prove are provided; the following example is illustrative: given a Banach space X and $p > 1$, defining $p_n := p - \frac{1}{\log(\log(n+4))}$, we prove the existence of a Banach space of maximal dimension formed, up to the null vector, by X -valued p -summable sequences not belonging to the Nakano space of X -valued sequences determined by $(p_n)_{n=1}^\infty$.

1. Introduction

The title *Linear subsets of nonlinear sets in topological vector spaces* of the recent survey [6] is a precise description of the field to which this paper belongs. It has been a long time that mathematicians have been searching for some linear structure in nonlinear environments, but the paper [2] and the works of Gurariy and his collaborators (see, e.g. [3; 14; 16; 17]) made the subject fashionable, and the long list of recent references in [6] ensures the vitality of the area. In this paper we continue the research initiated in [4; 8; 9] on the existence of infinite-dimensional closed subspaces of Banach or quasi-Banach sequence spaces formed by sequences with special properties. The state of the art is described in [6, Section 2.4.2], and very recent contributions can be found in [13; 15; 23].

Let us explain how our results push the results of [4; 9] quite further. Given a Banach space X , in [9] the authors introduce a large class of Banach or quasi-Banach spaces formed by X -valued sequences, called *invariant sequences spaces* (see Definition 2.1), which encompasses several classical sequences spaces as particular cases (see [9, Example 1.2]). Roughly speaking, the main results of [4; 9] prove that, for every invariant sequence space E of X -valued sequences and every subset Γ of $(0, \infty]$, there exist a closed infinite-dimensional subspace of E formed, up to the null vector, by sequences not belonging to $\bigcup_{q \in \Gamma} \ell_q(X)$

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