Michigan Math. J. 62 (2013), 665–671

Buser–Sarnak Invariants of Prym Varieties

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1. Introduction

Let $A = V/\Lambda$ be an abelian variety with a principal polarization H, viewed as a positive definite Hermitian form on the universal cover V. Recall that the Buser–Sarnak invariant (cf. [L2, Def. 5.3.1]) of (A, H) is defined as

$$m(A,H) := \min_{v \in \Lambda \setminus \{0\}} H(v,v).$$

In their seminal paper [BuSa] (see also [L2, Thm. 5.3.5]), Buser and Sarnak showed that if $A = J$ is the Jacobian of a curve with the natural polarization H_J then its Buser–Sarnak invariant is remarkably small:

$$m(J, H_J) \le \frac{3}{\pi} \log(4 \dim J + 3). \tag{1.1}$$

Their proof uses hyperbolic geometry of compact Riemann surfaces. A different approach to bounding $m(A,H)$ was discovered by Lazarsfeld ([L1]; see also [L2, Thm. 5.3.6]). He gave an upper bound on $m(A,H)$ in terms of the Seshadri number of (A,H), which is an algebro-geometric invariant. His result can be used to derive an upper bound on $m(A,H)$ depending on the algebro-geometric properties of $m(A,H)$. For Jacobians, this approach yields

$$m(J, H_J) \le \frac{4}{\pi} \sqrt{\dim J}, \tag{1.2}$$

which is much weaker than (1.1) but is still a nontrivial restriction for Jacobians. Bauer ([Ba]; see also [L2, Rem. 5.3.14]) used this approach to show that, for a Prym variety P with the natural principal polarization H_P,

$$m(P, H_P) \le \frac{4}{\pi} \sqrt{2 \dim P}. \tag{1.3}$$

Since (1.3) is an analogue of (1.2), it is natural to ask whether there is a better bound for $m(P, H_P)$ of logarithmic order in $\dim P$—that is, an analogue of (1.1). This question was raised explicitly by Bauer [Ba, p. 610].

In this paper, we use the work of [BPS] to answer this question as follows.

Theorem 1.1. *For a Prym variety* (P, H_P),

$$m(P, H_P) \le 2^{20} \log(2 \dim P).$$

Received September 4, 2012. Revision received October 9, 2012.
This work is supported by National Researcher Program 2010-0020413 of NRF and MEST.