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Plurisubharmonic Subextensions As Envelopes of Disc Functionals

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1. Introduction

The theory of disc functionals was founded just over twenty years ago. Its main goal is to provide disc formulas for important extremal plurisubharmonic functions in pluripotential theory, that is, to describe these functions as envelopes of disc functionals. This brings the geometry of analytic discs into play in pluripotential theory. Disc formulas have been proved for largest plurisubharmonic minorants (some of the main references, in historical order, are [14; 2; 15; 9; 19; 13]) and for various Green functions (see for example [16; 5; 10; 18; 12]). For recent generalizations to singular spaces, see [3; 4].

We continue this project by proving a disc formula for largest plurisubharmonic subextensions. Consider domains $W \subset X$ in complex affine space $\mathbb{C}^n$ or, more generally, in a Stein manifold. Let $\phi\colon W \to [-\infty, \infty)$ be upper semicontinuous, for example plurisubharmonic, and let $S\phi$ be the supremum of all plurisubharmonic functions u on X with $u|W \le \phi$. If X is covered by analytic discs with boundaries in W, then $S\phi$ is a plurisubharmonic function on X, the largest plurisubharmonic subextension of ϕ to X. Under suitable conditions on W and X, we prove that for every $x \in X$, $S\phi(x)$ is the infimum of the averages of ϕ over the boundaries of all analytic discs in X with boundary in W and center x (Theorems 3 and 4). In general, however, the disc formula can fail (Example 3).

A recent Stein neighborhood theorem of Forstnerič [6, Thm. 1.2] allows us to work with analytic discs that are merely continuous on the closed unit disc $\overline{\mathbb{D}}$. This is technically easier than the traditional approach that uses germs of holomorphic maps from open neighborhoods of $\overline{\mathbb{D}}$.

A new equivalence relation on the space $\mathcal{A}_X^W$ of analytic discs in X with boundary in W naturally appears in the proof of our disc formula. We call analytic discs in $\mathcal{A}_X^W$ *center-homotopic* if they have the same center and can be joined by a path in $\mathcal{A}_X^W$ of discs with that same center. The quotient of $\mathcal{A}_X^W$ by this equivalence relation, if it is Hausdorff, is a complex manifold with a local biholomorphism to X (Theorem 6). The sufficient conditions in Theorems 3 and 4 are naturally

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