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Absolute Chow–Künneth Decomposition for Rational Homogeneous Bundles and for Log Homogeneous Varieties

Jaya NN Iyer

1. Introduction

Suppose X is a nonsingular projective variety of dimension n defined over the complex numbers. Let $\mathrm{CH}^i(X) \otimes \mathbb{Q}$ be the Chow group of codimension-i algebraic cycles modulo rational equivalence, with rational coefficients. Murre [Mu2; Mu3] has made the following conjecture, which leads to a filtration on the rational Chow groups.

Conjecture. The motive $h(X) := (X, \Delta_X)$ of X has a Chow–Künneth decomposition:

$$\Delta_X = \sum_{i=0}^{2n} \pi_i \in \mathrm{CH}^n(X \times X) \otimes \mathbb{Q}$$

such that the π_i are orthogonal projectors (see Section 2.2).

In this paper, *absolute* Chow–Künneth decomposition (resp. projectors) is the same as Chow–Künneth decomposition (resp. projectors). We write "absolute" to emphasize the difference with the "relative" Chow–Künneth projectors that will appear in the paper.

Some examples where this conjecture is verified are: curves, surfaces, a product of a curve and surface [Mu1; Mu3], abelian varieties and abelian schemes [DenMu; Sh], uniruled threefolds [DM1], elliptic modular varieties [GHMu2; GMu], universal families over Picard modular surfaces [Mi+], and finite group quotients (which may be singular) of abelian varieties [AkJ], some varieties with nef tangent bundles [I], open moduli spaces of smooth curves [IM], and universal families over some Shimura surfaces [Mi].

In [I] we looked at varieties that have a nef tangent bundle. Given the structure theorems of Campana and Peternell [CP] and Demailly, Peternell, and Schneider [DePS], we know that such a variety X admits a finite étale surjective cover $X' \to X$ such that $X' \to A$ is a bundle of smooth Fano varieties over an abelian variety. Furthermore, any fibre that is a smooth Fano variety necessarily has a nef tangent bundle. It is an open question [CP, p. 170] whether such a Fano variety is

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