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Imprimitive Distance-Transitive Graphs with Primitive Core of Diameter at Least 3

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In memory of Donald G. Higman

1. Introduction

A distance-transitive graph G is one upon which the automorphism group acts transitively on ordered pairs of vertices at every fixed distance. Only connected graphs need to be considered. Those of diameter 2 are the rank-3 graphs, whose careful study was initiated by Donald G. Higman in his breakthrough paper [16].

A huge amount of effort has gone into the classification of all finite distance-transitive graphs. The classification naturally breaks into two parts, primitive and imprimitive. The main part of the problem is the classification of all finite distance-transitive graphs with primitive automorphism group, and it appears that this classification is nearly finished. For the imprimitive case, Smith [24] showed that the possibilities for nontrivial blocks of imprimitivity are severely limited and that a given imprimitive distance-transitive graph can in a sense be reduced to a primitive distance-transitive graph. Van Bon and Brouwer [5] and Hemmeter [14; 15] carried through the reverse of Smith's theorem, classifying for most of the known primitive distance-transitive graphs any associated imprimitive distance-transitive graphs they might have.

In [3] the present authors gave a precise version of Smith's theorem which implies that any unknown imprimitive distance-transitive graph must arise from a primitive distance-transitive graph of diameter at least 2 and valency at least 3. In the present paper, we return to the work of van Bon and Brouwer [5] and Hemmeter [14; 15] and show that, starting from each of the known distance-transitive graphs of diameter and valency at least 3, there are no surprises—the only associated imprimitive graphs are ones already known and in the literature (see [7]).

The terminology and results will be made precise in the next section. In particular we give a precise version of Smith's theorem (following [3]) and describe how the present results fit into the general problem of classifying all distance-transitive graphs. In Section 3 we give various general results about the parameters of a distance-regular graph, particularly those that are imprimitive in one of the two ways specified by Smith's theorem. Section 4 discusses various of the geometries, designs, and codes often used in constructing and describing the graphs under consideration. Of particular import are the gamma spaces introduced by Higman.

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