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On Abelian Coverings of Surfaces

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In this paper we consider only orientable compact topological surfaces without boundary, which for brevity are simply called *surfaces.* All autohomeomorphisms of surfaces are presumed to be orientation preserving.

We are interested in a classification of finite abelian coverings of surfaces up to the following equivalence relation: two coverings $\pi_1\colon T_1 \to S_1$ and $\pi_2\colon T_2 \to S_2$ are *equivalent* if there are homeomorphisms $\varphi\colon S_1 \to S_2$ and $\psi\colon T_1 \to T_2$ such that the diagram

$$\begin{array}{ccc} T_1 & \xrightarrow{\psi} & T_2 \\ {\scriptstyle \pi_1}\downarrow & & \downarrow{\scriptstyle \pi_2} \\ S_1 & \xrightarrow[\varphi]{} & S_2 \end{array}$$

commutes.

If $\pi: T \to S$ is a Galois covering with Galois group G (acting on T), then $S \simeq T/G$. Conversely, if G is a finite group of autohomeomorphisms of a surface T acting on T freely (i.e., with trivial stabilizers), then the factorization map $\pi: T \to T/G = S$ is a Galois covering with Galois group G.

Thus, instead of considering finite abelian coverings of surfaces one can consider pairs (T, G), where T is a surface and G is a finite abelian group of autohomeomorphisms of G acting on T freely. The foregoing equivalence relation for coverings corresponds to the following notion of isomorphism of pairs: two pairs (T_1, G_1) and (T_2, G_2) are *isomorphic* if there exist a homeomorphism $\psi: T_1 \to T_2$ and an isomorphism $f\colon G_1 \to G_2$ such that the diagram

$$\begin{array}{ccc} T_1 & \xrightarrow{\psi} & T_2 \\ {\scriptstyle g}\downarrow & & \downarrow{\scriptstyle f(g)} \\ T_1 & \xrightarrow[\psi]{} & T_2 \end{array}$$

commutes for any $g \in G_1$.

Given a finite abelian group G_0, one can consider the problem of classification of free G_0-actions on surfaces. This is not the same as classifying pairs (T, G) with $G \simeq G_0$. To each such pair there corresponds a set of isomorphism classes

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