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Convergence in Capacity of the Perron–Bremermann Envelope

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1. Introduction

In [CK1], Cegrell and Kołodziej constructed a sequence of measures in a ball μ_j converging to μ in the weak* topology such that the solutions of the Dirichlet problems

$$(dd^c u_j)^n = d\mu_j, \quad u_j = 0 \text{ on the boundary}$$

are uniformly bounded yet u_j does not converge to u, the solution of the Dirichlet problem

$$(dd^c u)^n = d\mu, \quad u = 0 \text{ on the boundary.}$$

In [CK2] the authors gave conditions on the Monge–Ampère mass of the solutions u_j, with fixed continuous boundary values φ, that guarantee the stability of the complex Monge–Ampère operator. They introduced the set $\mathcal{A}(\mu)$ of all solutions u of the Dirichlet problem $u \in \mathcal{F}(\varphi)$, $(dd^c u)^n = g d\mu$, where μ is a positive finite measure that does not put mass on pluripolar sets and where g varies over all μ-measurable functions satisfying $0 \le g \le 1$. Cegrell and Kołodziej proved that, in $\mathcal{A}(\mu)$, weak* convergence is equivalent to convergence in capacity.

Our main goal is to generalize this statement by admitting a large variation of the boundary data. Let Ω be a bounded domain in $\mathbb{C}^n$, let f be a bounded function on $\partial\Omega$, and let μ be a positive Borel measure on Ω. Following [BT1], we define

$$\mathrm{PB}(f,\mu) = \{u \in \mathcal{F}(g) : (dd^c u)^n \ge \mu,\ g \le f, \text{ and } g \text{ is upper semicontinuous on } \partial\Omega\}.$$

We shall refer to the following function as the *Perron–Bremermann envelope*:

$$U(f,\mu) = \sup\{v : v \in \mathrm{PB}(f,\mu)\}.$$

For a fixed positive finite measure μ that does not put mass on pluripolar sets and for a fixed positive constant k, we shall consider the family $\mathcal{D}(\mu,k)$ of plurisubharmonic functions $U(f, g d\mu)$, where g is μ-measurable function such that $0 \le g \le 1$ and where f varies over all upper semicontinuous functions on the boundary such that $|f| \le k$. We shall prove that, in $\mathcal{D}(\mu,k)$, pointwise convergence is equivalent to convergence in capacity.

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