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Deformation Theory of 5-Dimensional CR Structures and the Rumin Complex

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1. Introduction

A natural problem in several complex variables is that of classifying the deformations of an isolated singularity in a complex analytic variety. The problem is solved by constructing a "versal family" of deformations of the singularity, which is, roughly speaking, a minimal family of deformations that includes biholomorphic representatives of all other deformations. (See Section 8 for a precise definition.)

Versal families for isolated singularities were first constructed from an algebraic point of view in the late 1960s and early 1970s by Tjurina, Grauert, and Donin [Tj; Gr; D]. Shortly thereafter, Kuranishi [K] outlined a program for relating deformations of an isolated singularity to deformations of the CR structure on a real hypersurface obtained by intersecting the variety with a small sphere surrounding the singular point (the "link" of the singularity). His idea was to construct a versal family of deformations of the CR structure on the link (versal modulo "wiggles" of the link within the ambient complex space, not just modulo changes in the CR structure). Kuranishi's construction was extended and simplified by subsequent work of the first author and others [A3; A4; A5; M1; M2; BM].

A fundamental limitation of all of these results has been a dimensional restriction: Because the deformation complex that was introduced in [A3; A4; A5] failed to be subelliptic in low dimensions, these results applied only to CR manifolds of dimension 7 or more (and therefore to singularities of varieties whose complex dimension is at least 4).

The purpose of this paper is to extend the Kuranishi construction of versal families of CR structures to the case of 5-dimensional CR manifolds. The new idea here is a subelliptic estimate and consequent Hodge theory for a certain subcomplex of the standard deformation complex inspired by recent work of M. Rumin on contact manifolds.

Miyajima [M3], following an idea introduced in [Be], has introduced an alternative approach to constructing versal families in all dimensions that is based on analyzing deformations not only of the CR structure but also of the CR structure together with its embedding into $\mathbf{C}^N$. The present approach is of independent interest, however, because it represents a completion of the original Kuranishi

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