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Quartic 3-Fold: Pfaffians, Vector Bundles, and Half-Canonical Curves

A. Iliev & D. Markushevich

Introduction

This paper is a part of the study of moduli spaces of vector bundles with small Chern classes on certain Fano 3-folds. We investigate the moduli component of kernel bundles on a quartic 3-fold, defined similarly to that of [IM; MT] for the case of a cubic 3-fold. Our work received a strong pulse with the publication of Beauville's paper [B], which allowed us to simplify some arguments and put our results in a more general framework of Pfaffian hypersurfaces.

In [MT], it was proved that the Abel–Jacobi map of the family of normal elliptic quintics lying on a general cubic 3-fold V factors through a moduli component of stable rank-2 vector bundles on V with Chern numbers $c_1 = 0$ and $c_2 = 2$ and whose general point represents a vector bundle obtained by Serre's construction from an elliptic quintic. The elliptic quintics mapped to a point of the moduli component vary in a 5-dimensional projective space inside the Hilbert scheme of curves, and the map from the moduli component to the intermediate Jacobian is quasi-finite. In [IM], this moduli component was identified with the variety of representations of V as a linear section of the Pfaffian cubic in $\mathbb{P}^{14}$ and it was proved that the degree of the quasi-finite map is 1, so the moduli component is birational to the intermediate Jacobian $J^2(X)$. According to [D], the moduli space $M_V(2; 0, 2)$ is irreducible, so its unique component is the one just described.

In the present paper, we prove that a generic quartic 3-fold X admits a 7-dimensional family of essentially different representations as the Pfaffian of an 8×8 skew-symmetric matrix of linear forms. Thanks to [B], this provides a 7-dimensional family of arithmetically Cohen–Macaulay (ACM for short) vector bundles on X, obtained as the bundles of kernels of the 8×8 skew-symmetric matrices of rank 6 representing points of X. We show that this family is a smooth open set M_X in the moduli space of stable vector bundles $M_X(2; 3, 14) \simeq M_X(2; -1, 6)$. The ACM property means the vanishing of the intermediate cohomology $H^i(X, \mathcal{E}(j))$ for all $i = 1, 2$ with $j \in \mathbb{Z}$.

We also give a precise geometric characterization of the ACM curves arising as schemes of zeros of sections of the kernel vector bundles. According to Beauville, they are half-canonical ACM curves of degree 14 in $\mathbb{P}^4$; we show that they are linear sections of the rank-4 locus $Z \subset \mathbb{P}(\wedge^2\mathbb{C}^7)$ in the projectivized space of the

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