Spectral Synthesis of Ideals in Zygmund Algebras: The Asymptotic Cauchy Problem Approach

EVSEY DYN'KIN & LEONID HANIN

0. Introduction

In the present work, we study the structure of closed ideals in algebras of functions f on the cube $Q_0 := [-1, 1]^n$ satisfying, for a given majorant ω, the Zygmund condition $|f(x+h) - 2f(x) + f(x-h)| \leq C\omega(|h|)$. It is assumed that

$$\int_0^1 \frac{\omega(t)}{t^2}\, dt = +\infty \tag{0}$$

or equivalently that Λ^ω is not contained in $C^1(Q_0)$. The main result of the paper is a theorem on spectral synthesis of ideals in Zygmund algebras Λ^ω (Theorem 2) claiming that, for regular majorants ω (i.e., those subject to regularity conditions (R1) and (R2) below), every closed proper ideal in the algebra Λ^ω is an intersection of closed primary ideals. Assertions of such type go back to the classical algebraic works of E. Noether and E. Lasker on ideals in Noetherian rings. Later, theorems on spectral synthesis of ideals were proved (or disproved) for various algebras of smooth functions. For more extensive discussion, see Section 1.

Our proof of Theorem 2 depends on two major results: on an abstract spectral synthesis theorem for a class of functional Banach algebras, called the class of D-algebras and defined in terms of point derivations (Theorem 0) [H2; H4]; and on a special extension theorem for Zygmund functions (Theorem 1) which implies that the algebras Λ^ω are D-algebras.

The main difficulty in proving Theorem 1 arises from the absence of an intrinsic description of traces of Zygmund functions to general sets in $\mathbb{R}^n$ for $n > 2$. Using such descriptions for $n = 1, 2$ [Shv; H4; H5], direct proofs of Theorem 1 (and thereby of the spectral synthesis theorem) for algebras Λ^ω with *arbitrary* majorants ω satisfying condition (0) were obtained in [H3; H5]. An alternative proof of Theorem 1 in the case $n = 1$, $\omega(t) = t$, is presented in [H4].

Our proof of Theorem 1 is based on the method of quasiharmonic extensions of smooth functions [D1; D2]. This method works for any dimension n; however, it presupposes certain regularity of majorants ω.

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