# $p$-Dirichlet Energy Minimizing Maps into a Complete Manifold

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## 1. Introduction

Suppose $M$ is a compact $m$-dimensional $\mathcal{C}^2$ Riemannian submanifold of $\mathbb{R}^Q$ with (or without) boundary $\partial M$. Suppose $N$ is an $n$-dimensional $\mathcal{C}^2$ complete connected Riemannian submanifold without boundary of some Euclidean space $\mathbb{R}^P$ such that

$$N \subset N_\tau = \{y \in \mathbb{R}^P : \operatorname{dist}(y, N) < \tau(|y|)\}.$$

Here $N_\tau$ is a $\tau$-tubular neighborhood of $N$ in $\mathbb{R}^P$ such that the nearest point projection map

$$\pi : N_\tau \to N$$

(i.e., $\operatorname{dist}(y, N) = |y - \pi(y)|$ for all $y \in N_\tau$) exists and is as smooth as $N$, and

$$\tau : \mathbb{R}_+ \cup \{0\} \to \mathbb{R}_+$$

is a monotonically decreasing function.

The $p$-Dirichlet energy functional is the $L^p$-norm of the gradient defined on the admissible mapping space

$$L^{1,p}(M, N) = \{v \in L^{1,p}(M, \mathbb{R}^P) : v(x) \in N \text{ for } \mathcal{L}^m \text{ a.e. } x \in M\},$$

where $\mathcal{L}^m$ is the $m$-dimensional Hausdorff measure induced by the metric of $M$ and $1 < p \le m$. We say $u \in L^{1,p}(M, N)$ is $p$-Dirichlet energy-minimizing if

$$\int_M |\nabla u|^p \le \int_M |\nabla v|^p$$

for all $v \in L^{1,p}(M, N)$ in the same (relative) homotopy class of $u$.

Much work has been done regarding the existence and partial regularity of a $p$-Dirichlet energy-minimizing map, in particular the case when $p = 2$ (see [1] for references). For $\tau > c > 0$, White showed in [8] that there is such a minimizer among maps in the same $[p-1]$-homotopy class. Furthermore, if the image of a small ball of such a minimizer is contained in a compact subset of $N$, where $N$ may not be compact, then Hardt and Lin's [4] (or Luckhaus's

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