Invariant Subspaces of the Bergman Space and Some Subnormal Operators in $\mathbb{A}_1 \backslash \mathbb{A}_2$

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1. Introduction

Let D be the open unit disk in the complex plane $\mathbb{C}$. The Bergman space $L_a^2(D)$ consists of analytic functions in D with

$$\|f\|_{L_a^2} = \left(\int_D |f(z)|^2 \, dA(z) \right)^{1/2} < \infty,$$

where dA denotes the area measure in $\mathbb{C}$, normalized by a constant factor

$$dA(z) = dx\, dy/\pi.$$

Let μ be a finite positive Borel measure with compact support, and let $\operatorname{spt} \mu$ denote the support of μ. Let $P^2(\mu)$ denote the closure in $L^2(\mu)$ of analytic polynomials in z and let S_μ denote the operator of multiplication by z on $P^2(\mu)$. The operator S_μ is *pure* if $P^2(\mu)$ has no L^2 summand. A measure with support in the closed unit disk is a reverse Carleson measure for $L_a^2(D)$ if

$$\int_D |p|^2 \, dA \leq C \int |p|^2 \, d\mu$$

for every polynomial p. The set E denotes a compact subset of the unit circle $\mathbb{T}$ with positive Lebesgue measure. Set $\mathbb{T} \backslash E = \bigcup J_n$, where J_n is a connected component. We say that E satisfies the *Carleson condition* if

$$\sum_{n=1}^{\infty} m(J_n) \log \frac{1}{m(J_n)} < \infty,$$

where m stands for the normalized Lebesgue measure on $\mathbb{T}$, that is, $dm = (1/2\pi)\, d\theta$. Every closed arc of $\mathbb{T}$ satisfies the Carleson condition. It is easy to construct a nowhere dense subset of $\mathbb{T}$ satisfying the Carleson condition. Put

$$\mu_E = dA\,|_D + m\,|_E. \tag{1.1}$$

The *Cauchy transform* of a finite measure μ is defined by

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