On CR Mappings of Real Quadric Manifolds

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0. Introduction

According to the well-known theorem of Alexander [Al], each local CR diffeomorphism of the real unit sphere in $\mathbf{C}^n$, for $n>1$, extends to a complex rational mapping on all $\mathbf{C}^n$. This important result was generalized by several authors in different directions. Thus, Tumanov [Tu], Tumanov and Henkin [TH], and Forstnerič [Fo] transferred it to CR mappings of real quadric Cauchy–Riemann manifolds of an arbitrary codimension in $\mathbf{C}^n$. In the present note we generalize these results. Our main tool is the reflection principle due to Lewy [Le], Pinchuk [P1], and Webster [W1; W2; W3].

I express my thanks to S. Pinchuk for his interest in this work.

1. Result

We denote coordinates in $\mathbf{C}^n$ by (z, w), where $z \in \mathbf{C}^k$, $w \in \mathbf{C}^d$, $k+d=n$, and $k, d>0$. Let also $\langle z, \zeta\rangle = \sum_{j=1}^{k} z_j \zeta_j$. Consider in $\mathbf{C}^n$ a real generic manifold of the form

$$w_j+\bar{w}_j=\langle L_j(z), \bar{z}\rangle, \quad j=1, \ldots, d, \tag{1.1}$$

where each $L_j: \mathbf{C}^k \rightarrow \mathbf{C}^k$ is a $\mathbf{C}$-linear hermitian operator. To simplify the notation we shall write (1.1) in the form

$$w+\bar{w}=\langle L(z), \bar{z}\rangle, \tag{1.2}$$

setting

$$\langle L(z), \bar{z}\rangle=(\langle L_1(z), \bar{z}\rangle, \ldots, \langle L_d(z), \bar{z}\rangle). \tag{1.3}$$

A manifold M of the form (1.2) is said to be a *quadric* Cauchy–Riemann manifold (in short, quadric) of real codimension d in $\mathbf{C}^n$.

Let $T_p^c(M)$ denote the complex tangent space of M at $p \in M$. Recall that $T_p^c(M)=T_p(M) \cap i(T_p(M))$, where $T_p(M)$ is a real tangent space and $i=\sqrt{-1}$. The complex dimension of $T_p^c(M)$ does not depend on p in M, and is equal to k; it is called the *CR dimension* of M. The vector-valued (with values in $\mathbf{R}^d$) hermitian quadratic form (1.3), defined on $\mathbf{C}^k \cong\{(z, w): w=0\}=$

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