Extension Domains for A_p Weights

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1. Introduction

In this paper we determine conditions on a bounded domain D in $\mathbf{R}^N$ ($N \geq 2$) so that every A_p weight on D can be extended to an A_p weight on $\mathbf{R}^N$. We also give boundary conditions on a Jordan domain D in $\mathbf{R}^2$ so that D is an extension domain for A_p.

Let D be a connected open set in $\mathbf{R}^N$, $N \geq 2$. A positive, locally integrable function w on D is said to belong to the class $A_p(D)$, $1 < p < \infty$, if

$$\|w\|_p = \sup_{Q \in \mathfrak{F}_0} \left(\frac{1}{|Q|} \int_Q w\, dx \right) \left(\frac{1}{|Q|} \int_Q \left(\frac{1}{w} \right)^{1/(p-1)} dx \right)^{p-1} < \infty, \tag{1.1}$$

where $\mathfrak{F}_0$ denotes the set of all cubes contained in D. The class $A_p(D)$ has been extensively studied in the case $D = \mathbf{R}^N$; they are precisely the class of weights for which, for example, the Hardy–Littlewood maximal function

$$Mf(x) = \sup_{Q \ni x} \frac{1}{|Q|} \int_Q |f(t)|\, dt$$

satisfies

$$\int (Mf(x))^p w(x)\, dx \leq C_p \int |f(x)|^p w(x)\, dx.$$

There is a close connection between $A_p(D)$ and the space of functions of bounded mean oscillation, abbreviated BMO(D). We say $f \in \mathrm{BMO}(D)$ if

$$\sup_{Q \in \mathfrak{F}_0} \frac{1}{|Q|} \int_Q |f - f_Q| < \infty,$$

where $f_Q = (1/|Q|) \int_Q f\, dt$ denotes the average of f on Q. This connection is as follows. If $w \in A_p(D)$ then $\log w \in \mathrm{BMO}(D)$, while if $f \in \mathrm{BMO}(D)$ then (by the theorem of John and Nirenberg) $e^{\delta f} \in A_p(D)$ for some $\delta > 0$.

We say that the domain D is an *extension domain* for $A_p(D)$ if whenever $w \in A_p(D)$ there exists $W \in A_p(\mathbf{R}^N)$ such that $W = w$ a.e. on D. Extension domains for BMO are defined analogously and have been characterized in [6], where it is shown that D is an extension domain for BMO if and only if

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