CARTAN CONNECTIONS IN FOLIATED BUNDLES

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1. Introduction. Let M be a smooth connected manifold of dimension n and let $\mathfrak{F}$ be a smooth codimension q foliation of M. Let G be a Lie group and let H be a closed subgroup of G such that G/H has dimension q. Let $\pi: P \to M$ be a foliated principal H-bundle in the sense of [10]. We define a Cartan connection in P as a certain type of one-form ω on P with values in the Lie algebra of G. This generalizes the notion of a Cartan connection in an ordinary principal bundle and provides a unified setting for the study of Riemannian, conformal, and projective foliations as well as other types of geometric structures for foliations.

THEOREM 1. *Let ω be a complete Cartan connection in P. Then all the leaves of $\mathfrak{F}$ have the same universal cover. In particular, if $\mathfrak{F}$ has a compact leaf with finite fundamental group, then all the leaves of $\mathfrak{F}$ are compact with finite fundamental group.*

As a corollary to Theorem 1, we will obtain the stability theorem of B. Reinhart [23] for Riemannian foliations.

THEOREM 2. *Let ω be a complete flat Cartan connection in P. Let $p: \tilde{M} \to M$ be the universal cover of M and let $(G/H)^{\sim}$ be the universal cover of G/H. There is a locally trivial fiber bundle $\tilde{M} \to (G/H)^{\sim}$ whose fibers are the leaves of $p^{-1}(\mathfrak{F})$.*

As a corollary to Theorem 2, we will obtain the structure theorem of G. Reeb [22] for codimension one foliations of a compact manifold defined by a nonsingular closed one-form.

We consider projective and conformal foliations from the point of view of Cartan connections in foliated bundles and we obtain:

THEOREM 3. *Let $\mathfrak{F}$ be a complete projective or conformal foliation of codimension q ($q \geqslant 2$ in the projective case, $q \geqslant 3$ in the conformal case). If $\mathfrak{F}$ has a compact leaf with finite holonomy group, then all the leaves of $\mathfrak{F}$ are compact with finite holonomy group.*

THEOREM 4. *Let $\mathfrak{F}$ be a complete flat projective or conformal foliation of codimension q ($q \geqslant 2$ in the projective case, $q \geqslant 3$ in the conformal case) of a connected manifold M. Then the universal cover of M fibers over S^q, the fibers being the leaves of the lifted foliation.*

We give a class of examples of complete Cartan connections in foliated bundles (a class which includes the generalized Roussaire foliations) as well as an example in the incomplete case.

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