SURFACE SYMMETRY II

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Introduction. In Part I of this work [2] we presented a classification of the actions of a finite abelian group on a closed, orientable surface in terms of cobordism class and fixed point data. Here we extend those results to actions of certain nonabelian finite groups. In Section 1 we examine free actions of a split metacyclic group G, i.e., a semidirect product of two cyclic groups. The main result of Section 1 is as follows.

THEOREM. *Let G be a finite nonabelian split metacyclic group which acts freely on a closed, oriented surface M, preserving orientation. Then the set of orientation-preserving equivariant homeomorphism classes of free actions of G on M is in bijective correspondence with the second homology group $H_2(G; \mathbf{Z})$.*

Although this result is certainly of a rather specialized nature, we view it as somewhat remarkable that the free actions of a nonabelian group can be classified up to equivariant homeomorphism by the abelian group $H_2(G; \mathbf{Z})$.

Since $H_2(G; \mathbf{Z})$ can be identified with the free equivariant cobordism group $\mathcal{O}_2^{\text{free}}(G)$, we have the following consequence.

COROLLARY. *Two free orientation-preserving actions of a split metacyclic group on a closed oriented surface are equivalent, by an orientation-preserving equivariant homeomorphism, if and only if they are freely cobordant.*

The analogous statement for finite abelian groups G was the main result from [2].

PROBLEM. Find invariants other than $\mathcal{O}_2^{\text{free}}(G)$ for free G actions on surfaces.

In Section 2 we take a different tack and examine "indecomposable" actions which preserve no nontrivial family of disjoint simple closed curves. These actions have exactly three singular orbits and have orbit space the sphere. By *fixed point data* for an action we mean a description of the equivariant homeomorphism class of the restriction of the action to a neighborhood of the singular orbits. This can be described as an unordered set of conjugacy classes in G, with multiplicities allowed, one for each singular orbit. (See Section 2.) Recall that two G actions are *weakly equivalent* if they are equivalent modulo automorphisms of G. Similarly we say that two sets of fixed point data are weakly equivalent if there is an automorphism of G taking one onto the other.

THEOREM. *Two indecomposable actions of a split metacyclic group G on a surface M are weakly equivalent if and only if they have weakly equivalent fixed point data.*

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