ABSOLUTE VALUES OF HYPONORMAL OPERATORS WITH ASYMMETRIC SPECTRA

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1. Let H be an infinite dimensional, separable Hilbert space. Let the bounded operator T on H, with the Cartesian representation $T = A + iB$, be completely hyponormal, so that

$$T^*T - TT^* = D, \quad \text{equivalently,} \quad AB - BA = -\tfrac{1}{2}iD, \quad D \geqslant 0, \tag{1.1}$$

and T has no nontrivial reducing subspace on which it is normal. In addition, it will henceforth be supposed that T has a polar factorization

$$T = UP, \quad U \text{ unitary and } P = |T| = (T^*T)^{1/2}. \tag{1.2}$$

Such a factorization exists (and is unique) if and only if 0 is not in the point spectrum of T^*; see [9], p. 277.

Let A be a selfadjoint operator with the spectral family $\{E_t\}$. The set of vectors x in H for which $\|E_t x\|^2$ is an absolutely continuous function of t is a subspace, $H_a(A)$, and the operator A is said to be absolutely continuous in case $H = H_a(A)$. (See, e.g., Kato [2], p. 516.) Similar concepts can be defined for a unitary operator.

If T is completely hyponormal then its real and imaginary parts are absolutely continuous; further, if T has a factorization (1.2) then U is also absolutely continuous. (See [3], p. 42; [8], p. 193.) Simple examples show, however, that the absolute value of T, that is $P = |T| = (T^*T)^{1/2}$, need not be absolutely continuous or even have an absolutely continuous part. In fact, it has recently been shown ([1]) that any nonnegative operator P for which $\sigma(P)$ contains at least two points, $0 \notin \sigma_p(P)$, and for which neither $\max \sigma(P)$ nor $\min \sigma(P)$ belongs to $\sigma_p(P)$ with a finite multiplicity, is the absolute value of some completely hyponormal operator T with a factorization (1.2).

In case T is completely hyponormal and if the spectrum of $|T| = (T^*T)^{1/2}$ has Lebesgue linear measure zero (and whether or not T has a factorization (1.2)) then $\sigma(T)$ is radially symmetric. In fact, $\sigma(T)$ is the closure of a countable number, finite or infinite, of pairwise disjoint open annuli centered at the origin; see [6], p. 426. On the other hand, if T is completely hyponormal and if there exists some open wedge

$$W = \{z : z = re^{it},\ r > 0,\ -\pi < a < t < b \leqslant \pi\} \tag{1.3}$$

which does not intersect $\sigma(T)$, then necessarily $|T|$ is absolutely continuous. This follows from [6], p. 424, since, in the above case where $W \cap \sigma(T)$ is empty, necessarily T has a factorization (1.2). This last assertion follows from the fact

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