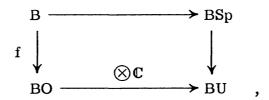
INDUCED COBORDISM THEORIES--AN EXAMPLE

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1. INTRODUCTION

The object of this note is to describe a way to construct new cobordism theories, basically by means of one example. First, recall that if M is a manifold, then the total space of the tangent bundle of M is an almost complex manifold, for $\tau(E(\tau_M)) \cong \pi^* \tau_M \oplus \pi^* \tau_M \cong \pi^* \tau_M \otimes_R \mathbb{C}$. If M is also a complex manifold, then τ_M is a complex bundle, so $\tau(E(\tau_M))$ is the complexification of a complex bundle and thus is a quaternionic (or symplectic) vector bundle.

One then introduces the notion of a weakly weakly almost complex manifold as a manifold together with a quaternionic structure on the complexification of the normal bundle. In bundle-theoretic terms, there is a fibering BSp \rightarrow BU obtained by considering a quaternionic bundle as just a complex bundle, and a map $\otimes \mathbb{C}$: BO \rightarrow BU obtained by classifying the complexification of the universal bundle. One may then form the induced fibering



and a weakly weakly almost complex manifold is a manifold M together with a chosen equivalence class of liftings of the normal map ν : M \rightarrow BO to B. (See Lashof [2] for the precise formalism of manifold with (B, f)-structure.)

Noting that the complexification of a complex bundle is quaternionic shows that

the composite BU $\xrightarrow{\pi}$ BO $\xrightarrow{\to}$ BU lifts to BSp, and hence every weakly almost complex manifold (for which ν lifts to BU) is weakly weakly almost complex.

Following Lashof, one may introduce the cobordism group $\Omega_*^{(B,f)}$ of weakly weakly almost complex manifolds. The main result of this paper is then:

THEOREM. The forgetful homomorphism $F\colon \Omega_*^{(B,f)} \to \mathfrak{R}_*$ into unoriented cobordism is monic, and one may choose generators x_i of $\mathfrak{R}_* = Z_2$ $[x_i \colon i \neq 2^s - 1]$ so that the image of F is the polynomial subalgebra on the x_i (i odd) and x_i^2 (i even).

Note. The image of the complex cobordism ring $\Omega^{\rm U}_*$ in \mathfrak{N}_* is the polynomial subalgebra consisting of the squares (Milnor [3]). The odd-dimensional generators needed may be taken to be U/O manifolds in the sense of Smith-Stong [4]; *i.e.*, manifolds for which the complexification of the normal bundle is trivial.

The results will include a general structure theorem (Remark following Lemma 3.1) showing that many theories are 2-torsion, and an analysis of Wall's cobordism theory W_{\star} (section 4) in a form similar to weakly weakly almost complex cobordism.

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